THE EFFECT OF STUDENT VERBALIZATIONS ON THE MATHEMATICAL PROBLEM SOLVING OF FOURTH GRADE STUDENTS WITH MATHEMATICAL LEARNING DISABILITIES (MLD)

By

Emmanuel Taiwo

A dissertation submitted to Johns Hopkins University in conformity with the requirements for the degree of Doctor of Education

Baltimore, Maryland
October 2017

Abstract

Students identified as having a mathematical learning disability (MLD) often struggle academically and are at risk of dropping out of school. Word problem solving can be particularly challenging for these students due to specific cognitive deficits that impact their conceptual understanding and procedural fluency as well as their overall mathematics achievement. This dissertation study employed a single case experimental study design with a multiple baseline approach to determine the effectiveness of verbalizations on the mathematical problem solving of four fourth grade students with MLD at a Public Charter School in an east coast metropolitan area. The students were randomly assigned to one of two special education teachers and participated in a mathematics intervention for up to twelve sessions of forty-five minutes each, not including the baseline data collection sessions. The visual and statistical analysis of data suggested a functional relation between student verbalizations and increase in mathematical proficiency (conceptual understanding and procedural fluency) of all the study participants. Using the standardized mean difference Cohen's d method, the effect sizes range from 0.77 to 1.73. Additionally, the researcher found a moderate positive effect on procedural fluency and a large positive effect on conceptual understanding for all the intervention recipients.

Keywords: mathematical learning disabilities (MLD), achievement gap, multiple baseline design, student verbalizations, word problem, conceptual understanding, procedural fluency

Dedication

I dedicate this dissertation to my loving and supportive wife, Olaitan, for her unswerving support and encouragements throughout my years in the doctorate program. I also dedicate this dissertation to our kind-hearted little girls, Esther and Elizabeth, and to my always encouraging, ever supportive parents, Paul and Bimpe Taiwo. All of you have nurtured my gifts and graciously pushed me in the right direction to achieve great things, thank you!

Acknowledgements

I wish to thank everyone that contributed to my success. Special thanks to my advisor, Dr. Juliana Paré-Blagoev, for her countless hours of proofreading, providing helpful feedback, and mentoring me throughout the entire process. I would also like to thank Dr. Karen Karp and Dr. Aaron Parsons for agreeing to serve on my dissertation committee. They offered many insightful recommendations and demonstrated a sincere interest in my work. I am grateful to have such a group of intelligent and respected researchers and teacher educators on my committee. Special thanks to Dr. Thomas Kratochwill for reviewing my initial conceptualization of the methodology and providing guidance on the appropriate research design.

I would like to thank my friends and colleagues at CCPCS. Specifically, I am very appreciative of the leadership of my school principal and executive sponsor, Amy Wendel. She believes in me and supports my professional growth and progress for many years. Also, some brilliant educators worked with me to conduct the needs assessment study and implement the instructional interventions associated with this dissertation study. Sincere thanks to Jessica Harrington, Yendry Quesada, Margo Brasil, and Devon Walter for agreeing to participate in the professional development and leading the instructional interventions. I would also like to thank Christopher Kenny, Judith Dorvil, LeShone HoSang Navis, Tsitsi Sithole, Jessica Curry, Michelle Fay, Charla Lewis, Christopher Ray and the lower school inclusion staff for encouraging me and contributing to the success of this project.

Within the Johns Hopkins doctor of education program, thanks to my friends in the 2014 cohort, especially Dr. Susanna Brock. She offered me support during the comprehensive exam, proposal defense, and dissertation defense. My story is incomplete without acknowledging my pastor for several years, Sunday Aworanti. He always motivates me to go further. He is an epitome of excellence. I offer the greatest thank you to my wife and best friend, Olaitan Taiwo. Thank you for the sacrifices you made so that I could complete this project. You single-handedly took care of our daughters, allowing me to focus on the doctoral studies. Your commitment to my success is unparalleled. Finally, my greatest gratitude goes to God, the source of my life and strength.

Table of Contents

List of Tables

List of Figures

CHAPTER 1: EXECUTIVE SUMMARY

Introduction

Students identified as having a mathematical learning disability (MLD) often struggle academically and are at risk of dropping out of school (Dunn, Chambers, & Rabren, 2004; Geary, 2004, 2007, 2011; NAEP 2015; Watson & Gable, 2013). Specifically, mathematical problem solving can be challenging for these students due to poor conceptual understanding and procedural fluency, as well as specific cognitive deficits that impact their overall mathematics achievement (Geary, 2004, 2007, 2011; Jitendra DiPipi, & Perron-Jones, 2002; Karagiannakis et al., 2014; Watson & Gable, 2012). Instructional practices and interventions aimed at improving the mathematical performance of students with MLD involve specific components such as explicit instruction, student verbalizations of their mathematical reasoning, use of visual representations while solving problems, repeated practice, and corrective feedback (Baker, Gersten, & Lee, 2002; Gersten, Jordan, & Flojo, 2005; Kroesbergen & Van Luit, 2003; Tournaki, 2003; Xin, Jitendra, & Deatline-Buchman, 2005).

Purpose of Study

The present study investigated the effects of verbalizations on the mathematical proficiency of four fourth graders with MLD. This study operationally defines students with mathematical learning disabilities (MLD) as elementary students with an Individual Educational Program (IEP) in mathematics with a designation of Specific Learning Disability (SLD) by the school/IEP team. This student subgroup represents about 7% of the total students (PK-12th) in the research setting. The study focuses on fourth grade students because identification of MLD usually begins in third grade (Fuchs, Fuchs, Powell, Seethaler, Cirino, & Fletcher, 2008) and these students would have received special education interventions for a year before participating in this study.

Theoretical Alignment

Verbalization strategies, which often involve verbally stating one's thinking processes while solving mathematical problems (Baker, Gersten, & Lee 2002; Gersten et al. 2008; Rosenzweig, Krawec, & Montague, 2011), are grounded in theoretical views of metacognition that emerge from the seminal work of Flavell (1979). Flavell (1979) defines the concept of metacognition as "thinking about thinking." In other words, metacognition is the knowledge about one's cognitive processes (Flavell, 1979; Veenman et al., 2006). It refers to the aspect of information processing that monitors, interprets, evaluates, and regulates the contents and processes of its own organization. Flavell's (1979) work laid the foundation for subsequent studies (Veenman et al., 2006; Geary, 2010) and confirmed the importance of student-mediated metacognitive processes as strong predictor for academic achievement and self-efficacy. Metacognition is a significant predictor of academic achievement in general, and mathematical performance in particular (Veenman, Van Hout-Wolters, & Afflerbach, 2006).

Literature Review

Two major bodies and foci of MLD research have emerged over the years: (a) the nature of MLD, and (b) the instructional interventions for children with MLD (Gersten et al., 2008). The researchers that studied the nature of MLD have extensively investigated (a) the underlying cognitive deficits (e.g., working memory deficits, slow processing speed, and difficulties with retrieval-based processes) associated with the mathematics difficulties often encountered by children with MLD; (b) subtypes and identification of MLD; (c) comorbidity with other disabilities; and (d) difficulties associated with assessment tools (Geary, 2011; Geary et al., 2000; Passolunghi & Siegel, 2004; Karagiannakis et al., 2014; Watson & Gable, 2013). On the other hand, the researchers that studied the instructional interventions for children with MLD have reported specific practices that improve the mathematics achievement of children with

MLD. These evidence-based practices include (a) explicit instruction; (b) student verbalizations of their mathematical thinking; (c) use of visual representations; (d) providing opportunities for repeated practice; and (e) providing timely and corrective feedback (Furlong, McLoughlin, McGilloway, & Geary, 2016; Gersten et al., 2009; Gersten & Clarke, 2007; Kroesbergen 2003). This dissertation study focuses on student verbalizations (student think-aloud) due to its large effect size for special education students (Gersten & Clarke, 2007; Gersten et al., 2009).

Methodology

This dissertation study employed a multiple baseline approach to determine the effectiveness of verbalizations on the mathematical problem solving of four fourth grade students with MLD at a Public Charter School in an east coast metropolitan area. The students were randomly assigned to two special education teachers and participated in mathematics intervention for up to twelve sessions of forty-five minutes each, not including the baseline data collection sessions. The students were taught how to verbalize their mathematical reasoning while solving mixed sets of addition, subtraction, multiplication, and division word problems using the THINK framework (Thomas, 2006). Since the multiple baseline approach requires staggering the introduction of the intervention across participants over time, the introduction of the independent variable (i.e., verbalization intervention) was staggered across the students in order to investigate changes in the dependent variables -- conceptual understanding and procedural fluency.

Results

The visual and statistical analysis of data suggested a functional relation between student verbalizations and increase in mathematical proficiency (conceptual understanding and procedural fluency) of all the study participants. Using the standardized mean difference Cohen's d method, the effect sizes range from 0.77 to 1.73. Additionally, the researcher found a moderate positive effect on procedural fluency and a large positive effect on conceptual understanding for all the intervention recipients.

Findings and Discussion

The study findings have several implications. At the broadest level, the study contributes to a relatively small body of research concerning the interventions for students with mathematical learning disabilities. It also broadens the scope of prior research by delineating the twin effects of student verbalizations on the conceptual understanding and procedural fluency of students with MLD. Despite the study limitations, there are some clear implications for instructional practices. For example, student verbalizations can be used to (a) determine specific areas of weakness in students' processing skills, (b) determine the source of student errors, and (c) enhance both procedural and conceptual understanding strategies to increase student's procedural fluency.

CHAPTER 1: THE ACHIEVEMENT GAP OF STUDENTS WITH MLD: A REVIEW OF LITERATURE

Introduction

The dissertation study focuses on mathematics underachievement among students identified as having a mathematical learning disability (MLD). This is a critical unmet need as students with MLD are more likely to exhibit mathematical difficulties and are at a greater risk of dropping out of school than any of the other student subgroups identified under the Every Student Succeeds Act (ESSA) of 2015 (Dunn, Chambers, & Rabren, 2004; Geary, 2004, 2007, 2011; NAEP 2015; Watson & Gable, 2013). Solving this problem is significant for the individual child, and has important implications at the school level as a school's failure to make Adequate Yearly Progress (AYP) has been partially associated with the performance of the special education subgroup (Eckes & Swando, 2009).

According to the NCLB mandate that expired in 2015, all student subgroups (with few exceptions) had to reach 100% proficiency by the 2013–2014 school year. In other words, students with MLD in the special education subgroup were expected to increase their proficiency levels at a faster rate than their general education peers in order to attain this proficiency goal. Fifteen years after NCLB, this dream failed to materialize, and students with MLD continue to score much lower than their peers without disabilities on state and national tests (NAEP, 2013; Kroesbergen & Van Luit, 2003; Deshler, Schumaker, Lenz, Bulgren, Hock, Knight, & Ehren, 2001). On December 10, 2015, President Obama signed into law Every Student Succeeds Act (ESSA), which reauthorized the Elementary and Secondary Education Act of 1965 (ESEA) and replaced the No Child Left Behind Act (NCLB) of 2002. Although ESSA eliminates AYP and the 100% proficiency requirement, the new law still mandates schools to ensure that every child has an equitable opportunity to succeed as well as access the general education curriculum.

Students with MLD demonstrate several academic deficits and behavioral challenges that adversely impact their performance on standardized testing and contribute to an achievement gap. For example, in 2015, only 16% of fourth graders with disabilities reached proficient or advanced on the National Assessment of Educational Progress (NAEP) test in mathematics, whereas 43% of fourth graders without disabilities achieved either proficient or advanced on the same test. In the metropolitan area where this study is being conducted, only 10% of fourth graders with disabilities reached proficiency on the NAEP test in mathematics, whereas the percentage of students without who performed at or above the NAEP proficient level was 31% in 2015. In the school setting for the current research, 26% of students with disabilities achieved proficient or above, and 51% of students without disabilities were at the level of proficient or above in mathematics on the district's standardized assessment in 2014. While this site is above the district or national percentages they are still distressingly low.

In spring 2015, the school and others in the city participated in the Partnership for Assessment of Readiness for College and Careers (PARCC) assessments for the first time. Results from this new assessment also provides evidence of the mathematics achievement gap of elementary students with MLD. For instance, on the 2015 PARCC mathematics assessment, no (0%) 4th grader with MLD at the school met the expectations (i.e., achieved proficiency) for grade-level mathematics standards. On the other hand, 17.3% of students without MLD scored at or above proficient on the same mathematics test. In 2016, only 10% of 4th graders with MLD at the school achieved proficiency in mathematics; whereas, 29.4% of students without MLD met or exceeded expectations for grade-level mathematics standards or achieved proficiency in mathematics. The local data on multiple assessments document a gap between students with and without MLD. It is further disheartening that the achievement gap between elementary children with and without disabilities widens every year they are in school because schools do not appropriately meet the needs of this student subgroup (Deshler et al., 2001).

The seminal work of Coleman (1966) brought attention to the academic achievement inequity and gap among students. Since then, the achievement gap dilemma has generated enormous debates and resulted in a significant body of research and investigations (Coleman et al., 1966; Guo & Harris, 2000; Lee, 2012; Oakes and Rossi, 2003; Sirin, 2005; von Hippel, 2009). The large-scale standardized tests and reliable source of data for examining the achievement gap of students is the National Assessment of Educational Progress (NAEP). According to NAEP, an achievement gap occurs when one group of students significantly outperforms another group. In policy and practice, the term "achievement gap" often denotes the differences between the test scores of two or more student subgroups.

For several decades, researchers have primarily examined the achievement gap between minority and White populations; however, little attention has been paid to the achievement gap between students with and without disabilities (Byrnes, 2003; Lee, 2002). Other student subgroups experiencing achievement gaps include the English language learners, students from low-income families, and students from culturally and linguistically diverse backgrounds. This dissertation study only addresses the achievement gap of students with learning disabilities in mathematics.

Several studies have revealed the short- and long-term costs and economic implications of the underachievement of students with MLD. For example, early mathematical deficits are associated with and can lead to lifelong economic struggles (Geary, 2011), increased risk of dropping out of school and lack of academic engagement during the high school years (Reschly & Christenson, 2006), and unemployment, variance in employment, income, and lower work productivity (Gross, Hudson, & Price, 2009; Fuchs et al., 2008). The achievement gap needs urgent attention in order to address these significant economic repercussions which reflect a waste of human resources at the individual, family, community, and national levels.

Purpose of the Study

There is an extensive literature on interventions for struggling readers; however, there has been comparatively little literature published on mathematical learning disabilities. For example, ERIC and PubMed databases searches by Gersten, Clarke, and Mazzocco (2007) revealed quantitative evidence of a startling discrepancy between the number of research studies on reading disabilities as compared to the number of studies on mathematics disabilities. They found that the ratio of studies on reading disabilities to mathematics disabilities for the period 1996–2005 was 14:1. When compared to the evidence available through reading disabilities research, few researchers have investigated the nature of mathematical learning disabilities or the strategies and interventions used. The present study sought to contributes to a relatively small body of research concerning the interventions for students with MLD.

This chapter aims to describe the concept, history, and subtypes of MLD as well as the factors and underlying causes associated with the achievement gap of students with MLD. The following questions guided the search of the existing research:

- **Q1**: What is the history of mathematical learning disabilities (MLD)?
- **Q2:** What are the accepted definitions and eligibility criteria for mathematical learning disabilities (MLD)?
- **Q3:** What factors and underlying causes are associated with the achievement gap of children with mathematical learning disabilities (MLD)?

History of Mathematical Learning Disabilities (MLD)

The history of the examination and identification of mathematical learning disabilities (MLD) shows the influence of investigations conducted in the fields of medicine (neurology), developmental psychology, cognitive science, mathematics education, special education, and law (Gersten, Clarke, & Mazzocco, 2007). While discussing the historical and contemporary perspectives on MLD, Gersten, Clarke, and Mazzocco (2007) mention that Lewandowsky and Stadelmann reported a case in 1908 that established that mathematical difficulties (now considered as MLD) were associated with a legion on the brain's left hemisphere. Two decades later, a Swedish neurologist, Henschen, conducted the first systematic medical study of arithmetic disorder (now known as MLD). Henschen's research and other groundbreaking work in the field of medicine revealed and established mathematics as a complex, cognitive, and biological construct (Gersten et al., 2007). The work of these early medical scientists paved the way for future research in neuroscience that investigated mental and cognitive processes associated with the learning of mathematics. It also highlighted more broadly how the study of atypical performance can influence and inform our understanding of normal development and function in children.

In 1970, Ladislav Kosc conceptualized developmental dyscalculia (now MLD) as a "structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of mathematical abilities adequate to age, without a simultaneous disorder of general mental functions" (1970, p. 192). While discussing the framework for identifying children with developmental dyscalculia, Kosc (1970) proposed the IQ-discrepancy model. This model only identifies individuals with a substantial discrepancy between their demonstrated mathematical ability and their expected performance based on measured general intelligence or mental abilities. However, this is not without controversy as Fletcher, Morris, and Lyon (2003) have reported widespread frustration with the IQ-discrepancy model. Some frustrations stemmed from the fact that some of the children who did not meet IQ-discrepancy criteria may indeed have learning disabilities and vice versa. Therefore, Fletcher et al. (2003) contended that a discrepancy model should not be one of the defining features of identifying students with MLD. It is important to note that Kosc's work informed the creation of the special education legislation entitled Education for All Handicapped Children Act (EHA) in 1975.

Gersten and colleagues (2007) noted that the 1980s were a time of significant progress and breakthrough in MLD in particular from the field of cognitive psychology and within this field especially from work done to develop the information processing and cognitive processes theories. Efforts by Pellegrino and Goldman (1987) were particularly influential and advanced our understanding of the effective strategies for teaching students with MLD. For example, Pellegrino and Goldman (1987) found that students who struggle with MLD were unable to automatically store and retrieve basic arithmetic facts (e.g., $4 + 3 = 7$ or $4 \times 3 = 12$). Hence, the researchers established that automaticity with math facts was critical and it could be improved by systematic and extended drill and practice. Hasselbring and colleagues (1988) investigated effective ways to teach mathematics to students with MLD using technology and reported that computer-assisted instruction could enhance the automaticity of addition facts for students with MLD. This approach allows children with MLD to store arithmetic facts in memory as well as retrieval the information "quickly, effortlessly and without error" (p. 2). Recent studies by Mohd Syah, Hamzaid, Murphy, and Lim (2016), Bryant et al. (2015), and Salminen et al. (2015) established that technology-mediated interventions which make use of these types of findings can be effective at improving the performance of students with learning disabilities in mathematics.

In the early 1990s, Geary and other researchers investigated the cognitive, neuropsychological, and genetic correlates of mathematical achievement and mathematical learning disabilities. Geary's (1993) work established that deficits in working memory significantly contribute to mathematics struggles experienced by children with MLD. Geary (1993) claims that working memory is a principal factor associated with the poor mathematics ability of children with MLD. Several studies have supported Geary's (1993) work and contributed to our understanding of the cognitive mechanisms associated with the MLD.

In the early 2000s, several researchers attempted to describe MLD subtypes. For example, based on number knowledge deficits, Geary (2004, 2005) proposed three key subtypes of MLD: (a) procedural (left hemisphere), (b) semantic memory (left hemisphere), and (c) spatial (right hemisphere). However, Desoete (2007) and Karagiannakis, Baccaglini-Frank, and Papadatos (2014) did not find these three subtypes useful because the "profiles of the children met in practice do not appear to belong to any subtype" (Karagiannakis et al., 2014, p. 3). Later, the researchers posited a classification model for MLD consisting of four domains: core number, visual-spatial, memory, and reasoning. The subtypes are briefly discussed in the subsequent section of this chapter.

Researchers from the field of educational psychology contributed immensely to the furtherance of MLD research in the millennium. Researchers such as Jordan and colleagues (Hanich & Jordan, 2001; Jordan, Kaplan, & Hanich, 2002) investigated the relationship between reading disabilities (RD) and mathematical learning disabilities (MLD). For example, Jordan, Kaplan, and Hanich (2002) conducted a longitudinal study of mathematical competencies in 180 children (age range between 7 and 9 years) with specific mathematics difficulties versus children with comorbid mathematics and reading disabilities. Among other findings, Jordan et al. (2002) reported that: (a) children with comorbid RD and MLD may differ in characteristic ways from their peers who are impaired in either RD or MLD; (b) children with RD can be separated from children with MLD on cognitive measures; (c) math fact retrieval is a major deficit in children with MLD; and (d) children who have MLD but read on grade level, mathematically outperform their peers with comorbid mathematics and reading disabilities or just reading disabilities.

Although MLD research is in its infancy (Gersten, Jordan, & Flojo, 2005), two major bodies and foci of research have emerged over the years: (a) the nature of MLD, and (b) the instructional intervention for children with MLD (Gersten et al., 2008). For instance, Fuchs and colleagues (Fuchs, Fuchs, Powell, Seethaler, Cirino, & Fletcher, 2008; Fuchs, Fuchs, & Prentice, 2004) extensively work on mathematics instruction and interventions for students with MLD. Other researchers such as Baker, Gersten, and Lee (2002), Gersten et al. (2008), and Kroesbergen and van Luitt (2003) conducted meta-analyses in order to synthesize the research on instructional approaches and interventions that improve the mathematics achievement of children with MLD. Finally, MLD research has advanced due to input from multidisciplinary and transdisciplinary efforts. Looking ahead, Gersten and colleagues (2007) noted a "renewed interests in the concept of working memory, its influence, and its potential to play a pivotal role in examining and understanding MLD" among MLD researchers. (p. 23).

Theoretical Framework

Information processing theories focus on the processes and mechanisms through which learning occurs (Schunk, 2012) and have significantly informed our understanding of MLD (Geary, 2005). A guiding framework of this approach is that the human mind works like a computer or information processor. Using this framework, cognitive psychologists have investigated the processes and structures that underlie cognitive performance of children and there is consensus that: (a) information (input) from the environment is processed by a series of cognitive processing systems (e.g., attention, perception, and memory); and (b) the processing systems influence the information in systematic ways (Butterworth, 2010; Geary, 2010; Johnson et al., 2010; Peng et al., 2016; Raghubar, Barnes, & Hecht, 2010; Swanson & Sachse-Lee, 2001; Wang, Fuchs, & Fuchs, 2016).

The popular model of information processing theory includes three components: sensory memory, short-term memory, and long-term memory. Figure 1 shows these three elements and how children acquire, process, store, and retrieve information. The cognitive processing deficits associated with MLD impact mathematics achievement (Johnson et al., 2010; Raghubar, Barnes, & Hecht, 2010; Swanson & Sachse-Lee, 2001; Wang, Fuchs, & Fuchs, 2016). It is important to note that the sequential or linear format (i.e., input-processing-output) of the information processing model discussed by Atkinson and Shiffrin (1968) shown below has been criticized by some researchers because it presents a simplistic view of the human brain. Further work has been conducted to elaborate on and unpack the implied cognitive processes.

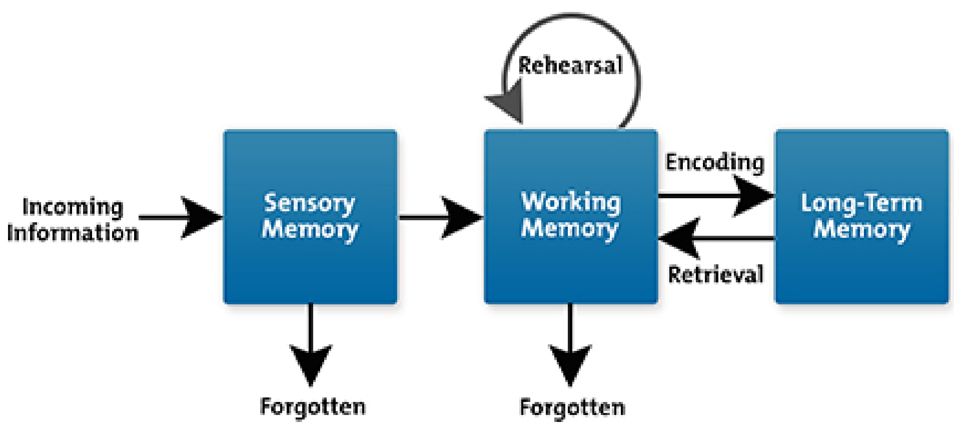


Figure 1. Information processing model (Atkinson and Shiffrin, 1968).

Over the years, MLD researchers have established that learning mathematical concepts involves the acquisition of both conceptual understanding and procedural skills. Such learning involves multiple cognitive processes including executive function (e.g., shifting, inhibition), working memory, phonological processing, and others (Geary, 2010; Johnson et al., 2010; Wang et al., 2016; Watson & Gable, 2013). While the full complexity is not well characterized, working memory (WM) has been implicated as a principal factor in children with MLD (Gersten et al., 2007; Swanson, 2006; Watson & Gable, 2013). In this regard, the present study acknowledges WM as a major cognitive process that underlies the mathematical problems experienced by children with MLD. And, in fact, WM has been found to be a reliable predictor of students' problem-solving abilities (Geary, 2010; Fuchs et al., 2008; Johnson et al., 2010; Swanson, 2006; Wang et al., 2016; Watson & Gable, 2013).

Fuchs and colleagues (2008) investigated the effect of cognition on the problem-solving and computational skills of children. The authors reported that student performance on computational tasks is associated with working-memory problems. The adverse impact of working-memory problems is profound on the mathematic performance of children with MLD. For example, Hasselbring, Goin, and Bransford (1988) worked with 160 "mildly handicapped and nonhandicapped students" aged 7-14 to investigate the impact of computer-based drill and practice on their mathematics automaticity. The researchers found that a typically achieving 12-year-old performed three times better than same aged children with MLD in mathematics tasks that involve fluent retrieval of basic math facts. Their work built the foundations for subsequent research studies that affirm the fact that children with MLD may have processing deficits that impede their ability to process, store, and retrieve relevant information.

Similarly, in the 1990s, Geary and colleagues investigated the cognitive mechanisms that influence the achievement of children with MLD. The researchers reported that children with MLD experienced difficulties retrieving basic math facts. According to cognitive and information processing theorists, children with MLD have limited cognitive capacity which makes it difficult for them to retrieve information (Pellegrino & Goldman, 1987). Most students with MLD have procedural knowledge of basic math facts (i.e., they could correctly add 4 and

5); however, they lack the ability to store the math facts in memory in a way that allows for retrieval "quickly, effortlessly and without error" (Hasselbring et at., 1988, p. 2).

Definitions of Mathematical Learning Disabilities (MLD)

Defining mathematical learning disabilities (MLD) is no easy task. The term mathematical learning disabilities (MLD) has had many names and connotations in the past: acalculia, dyscalculia, arithmetic disorder, mathematics disorder, specific learning disability (SLD) in mathematics, mathematical learning disabilities (MLD), mathematical disabilities, and mathematics difficulties. There is no consensus across different domains on appropriate terms to describe MLD (Mazzocco, 2007). The different terminologies and inconsistent meanings reflect challenges associated with historical context, the unclear nature of the disorder, and inconsistent diagnostic criteria used across disciplines in studying the phenomenon (Mazzocco, 2007). It is important to note that this chapter does not include an exhaustive list of MLD terminologies and definitions; however, notable contemporary definitions used by researchers and practitioners are summarized. Some of the most commonly used biologically based termis are mathematical disabilities, mathematical learning disabilities, mathematical difficulties, and dyscalculia. Researchers use these terms interchangeably when referring to children experiencing mathematical difficulties due to a disorder characterized by specific cognitive deficits.

The present study adopts the term mathematical learning disabilities (MLD) (Berch & Mazzocco, 2007) as well as the definition and eligibility guidelines provided by The Individuals with Disabilities Education Act (IDEA) in 2004. The IDEA is a United States law ensuring adequate support and services to children with disabilities throughout the nation. In order to meet the definition (and eligibility for special education and related services) as a “child with a disability,” a child's educational performance must be adversely affected due to the disability. As a result, the states must provide a free appropriate public education (FAPE) to any child with a disability. The initial regulations for SLD were finalized on December 29, 1977. The research and theories of Kosc (1970) heavily influenced this special education legislation.

According to IDEA, a mathematical learning disability (also referred to as Specific Learning Disability (SLD) in mathematics) is "a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may show itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations…" (IDEA, 2004). Further, the term does not include "learning problems that are primarily the result of visual, hearing, or motor disabilities; of intellectual disability; of emotional disturbance; or of environmental, cultural, or economic disadvantage." *The Diagnostic and Statistical Manual of Mental Disorders* of the American Psychiatric Association defined MLD as a discrepancy between performance on mathematics achievement tests and expected performance based on age, intelligence, and years of education.

An important term that is worthy of discussion, although it embodies a wider construct, is mathematical difficulty. Several research studies define children with mathematical difficulties as those whose standardized mathematics achievement falls below a cutoff score of approximately the 35th percentile. Other researchers maintain a different position in regards to the criteria for MLD and mathematics difficulties (low achieving) (Berch & Mazzocco, 2007; Geary, 2011; Mazzocco, Hanich, & Early, 2007). Geary (2011) noted that school-age children who score at or below the 10th percentile on standardized mathematics achievement tests for at least two consecutive academic years are usually categorized as MLD in research studies, and children scoring between the 11th and the 25th percentiles in at least two consecutive years are categorized as low achievement students. Murphy et al. (2007) reported cutoff criteria ranging from the 10th to 35th percentile, some times higher in cognitive research studies, up to the 45th percentile. Also, researchers advocate a more stringent cutoff, that is, the 10th percentile because it is more consistent with the international prevalence rates for MLD.

Although the behavioral or performance manifestations from distinct cognitive mechanisms may in fact be the same, the cognitive mechanisms causing these deficits may differ (Mazzocco, 2007; Watson & Gable, 2013; Geary, 2011). A key distinction between mathematical difficulties and mathematical learning disabilities (MLD) is that the latter is linked to cognition deficits as well as biological and neurological factors (Mazzocco, 2007). Kosc (1974) coined the word pseudo-dyscalculia, a term used to describe mathematical difficulties that resulted from inadequate or inappropriate instruction. Some students who struggled in mathematics may not have MLD, but rather experienced delays and difficulties in mathematics due to inadequate academic instruction (pseudo-dyscalculia) or having had limited number awareness at the time they entered school (Clarke et al., 2011; Geary, 2011). Further, Shalev, Manor, and Gross-Tsur (2005) discuss other health, environmental, and social factors that may "masquerade" as either learning disabilities or have an effect on the learning situation. These include, but are not limited to, ADHD, mathematics anxiety, overcrowded classes, mainstreaming of children with different capabilities, inadequate teaching methods, untested curricula, emotional issues, and family adversity/poverty.

Prevalence and Classification of Mathematical Learning Disabilities (MLD)

The National Center for Education Statistics (NCES, 2016) reported that the number of school-age children and youths receiving special education services was 6.6 million in 2014–15, this represents approximately 13% of total public school enrollment. Among these students, nearly 36% are eligible for specialized instruction and related services under the specific learning disabilities (SLD) category. Approximately 7% of children have learning disabilities in mathematics, and another 10% are considered "at risk" or "low achievers" (Geary, 2011;

Murphy, Mazzocco, Hanich, & Early, 2007; Shalev, Manor, & Gross-Tsur, 2005). The diagnostic criteria used for identifying children with MLD are inconsistent across the nation, and this has led to the misidentification of some students in the special education program. For example, Mazzocco and Myers (2003) found that "approximately 30% of primary-school-age children who met MLD criteria in one grade failed to meet these criteria a second time by third grade" (p. 40). Likewise, Schulte and Stevens (2015) contend that existing identification criteria for identifying children with learning disabilities may be biased. The researchers found that "even students who exit special education continue to be at risk for lower mathematics achievement" (p. 370). The findings suggest that the present approaches and criteria used in exiting students from special education and related services are not adequate.

There is no consensus regarding the classifications and subtypes of MLD (Karagiannakis et al., 2014; Mazzocco, 2007; Watson & Gable, 2013). Karagiannakis and colleagues (2014) assert that no single framework or model completely reflect the mathematical difficulties experienced by children with MLD. Nonetheless, using a neurocognitive approach, Karagiannakis and colleagues (2014) developed a four-dimensional classification model of MLD. The model identifies four cognitive domains within which specific deficits may reside: (a) core number (mathematical difficulties that impact only arithmetic domain), (b) retrieval and processing (memory difficulties that impact all mathematics domains), (c) reasoning (various executive mechanism deficits that impact all mathematics domains), and (d) visual-spatial subtype associated with difficulties in the areas of written arithmetic, geometry, algebra, analytical geometry, and calculus).

Paucity of Mathematical Learning Disabilities (MLD) Research

Several researchers have documented the long-term effect of mathematics proficiency or limitations to that proficiency. Duncan et al. (2007) found that the overall academic achievement of students is best predicted by their mathematical skills and the next best prediction is by reading and language skills. Geary (2011) discusses the long-term economic consequences of poor mathematics achievement. The author argued that the "social and individual costs of poorly developed mathematical skills may be higher than those associated with poor reading skills, in part because more people have difficulty with mathematics than with reading and because of steady increases in the quantitative knowledge needed to function in many jobs today, including many blue-collar jobs.” (p. 1).

Despite the consequences of poorly developed mathematical skills, MLD research was often treated as an afterthought in special education research (Gersten et al., 2008). Even though one would expect more research on mathematics interventions considering its immediate and long-term impact; regrettably, little attention has been paid to the field of MLD in this area of study. On the other hand, a considerable amount of literature has been published on interventions for reading disabilities (RD) (Geary, 2011; Gersten, Clarke, and Mazzocco, 2007; Gersten, Jordan, & Flojo, 2005; Mazzocco & Myers, 2003). Despite the paucity of MLD research, the last decades have brought steady research progress and increased the understanding of the diagnosis, remediation and prevention of MLD. Looking ahead, researchers need to delineate the developmental trajectories of MLD related to learning mathematics, identify the neural basis of the difficulties experienced by children with MLD, and develop appropriate interventions that treat the difficulties.

Factors and Underlying Causes Associated with MLD

Researchers have established multiple theories and factors associated with the mathematics difficulties encountered by children with MLD. Some of the factors and underlying causes associated are examined below through multiple theoretical perspectives and multidisciplinary lenses such as psychology, neuro- and cognitive science, medical genetics, and socioeconomics. This section emphasizes the cognitive factors and processes above other factors because of their prevalence in MLD research. A considerable amount of literature has established that working memory deficits, slow processing speed, and difficulties with retrieval-based processes are some of the cognitive factors associated with underachievement of children with MLD (Karagiannakis et al., 2014; Geary, 2011; Passolunghi & Siegel, 2004; Geary et al., 2000). In other words, cognitive processes have been implicated as being responsible for mathematics difficulties (See Table 1) experienced by children with MLD. Other factors include class size, teacher quality, teacher attrition, funding, among others. Chapter two describes the needs assessment study used to examine and refine the underlying causes and factors as they manifest within the school where the research is being conducted.

Table 1

MLD subtypes, cognitive systems involved, and typical mathematical difficulties encountered

MLD Subtypes	**Specific systems involved**	**Mathematical difficulties**
Core number	Internal representation of quantity: • ANS • OTS • Numerosity-Coding • Representation of symbols • Access deficit	Arithmetical domain: 1. Basic sense of numerosity and estimating accurately a small number of objects e.g., 4–5 (subitizing). 2. Estimating different quantities. 3. Placing numbers on number lines. 4. Managing Arabic symbols. 5. Transcoding a number from one representation to another (analog-Arabic-verbal). 6. Grasping the basic counting principles. 7. Capturing the meaning of place value (including in decimal notation). 8. Capturing the meaning of the basic arithmetic operation symbols (+, −, ×, ÷).
Memory (retrieval and processing)	• Working memory (WM) • Inhibition of irrelevant information from entering WM	All mathematical domains: 1. Retrieving numerical facts. 2. Decoding—confusing terminology (numerator, denominator, isosceles, equilateral). 3. Transcoding verbal rules or orally

	• Semantic memory	presented tasks. 4. Performing mental calculation accurately. 5. Remembering and carrying out procedures as well as rules and formulas. 6. (Arithmetic) problem solving (keeping track of steps).
Reasoning	Various executive mechanisms: • Entailment • Inhibition (not connected to WM) • Updating relevant information, shifting from one operation-strategy to another • Updating and strategic planning • Decision-making	All mathematical domains: 1. Grasping mathematical concepts, ideas and relations. 2. Understanding multiple steps in complex procedures/algorithms. 3. Grasping basic logical principles (conditionality — "if… then… " statements—commutativity, inversion). 4. Problem solving (decision making).
Visual- Spatial	• Visuo-spatial (VS) WM • Visuo-spatial reasoning/percepti on	Domains of written arithmetic, geometry, algebra, analytical geometry, calculus: 1. Interpret and use spatial organization of representations of mathematical objects (for example, numbers in decimal positional notation, exponents, or geometrical figures) 2. Placing numbers on a number line. 3. Recognizing Arabic numerals and other mathematics symbols (confusion in similar symbols). 4. Written calculation, especially where position is important (e.g., regrouping) 5. Controlling irrelevant visuo-spatial information. 6. Visualizing and analyzing geometric figures (or subparts of them), in particular visualizing rigid motions such as rotations. 7. Interpreting graphs, understanding and interpreting when the math information is organized visual-spatially (tables).

Note. The classification model for MLD was adapted from Karagiannakis, Baccaglini-Frank, and Papadatos (2014, p. 2). The researchers described four basic cognitive domains with the deficits causing mathematical difficulties for children with MLD.

Neurobiological factors related with MLD. Mazzocco (2001) examined whether indicators of MLD were observed in 5- and 6-year-olds with neurofibromatosis type 1 (NF1) and Turner syndrome or fragile X syndrome. The researcher submits that the underlying cognitive mechanisms leading to MLD may be enhanced by the study of genetic syndromes (i.e., Fragile X and Turner syndrome) connected to poor mathematics performance. Mazzocco (2001) found that individual differences in mathematical abilities are in part due to genetic and environmental factors, demonstrating, for example, a strong familial transmission of dyscalculia (MLD). Individuals with dyscalculia (MLD) may have poor comprehension of symbols, struggle with memorizing and organizing numbers, exhibit difficulty telling time, or have trouble with counting. Data suggest that nearly half of siblings of dyscalculics are dyscalculic, representing 5-10 times greater risk than controls (Shalev et al., 2001). Few other studies consistently indicate that brain abnormalities in children with MLD are probably of a genetic origin (Ansari, 2008). On the contrary, Del'Homme and colleagues (2007) found no familial association for MLD based on data from twin and family studies.

Cognitive factors related with MLD. Learning mathematics involves many different cognitive processes and systems such as working memory, executive function (e.g., shifting, inhibition), perceptual reasoning, phonological processing, visual or spatial skills, (Bull & Scerif 2001; Geary et al., 2007; Gersten et al., 2005; Geary et al., 1999; Passolunghi & Siegel, 2004; Karagiannakis et al., 2014; Watson & Gable, 2013). The current challenge is that the intricacy of interactions among these cognitive processes is yet to be fully understood, and we have a limited understanding of the cognitive processes associated with MLD (Watson & Gable, 2013). Yet, as noted earlier, several studies have implicated impairments of memory (e.g., working memory) underlie the mathematical problems and difficulties experienced by children with MLD. Working memory is often described as the information available in an easily accessible state that aids the completion of cognitive tasks (Cowan, 2010). Working memory is the most reliable indicator of MLD (Gersten et al., 2005; Karagiannakis et al., 2014; Watson & Gable, 2013).

Working memory is often conceptualized as the information available in an easily accessible state that aids the completion of cognitive tasks (Cowan, 2010). It helps the students to remember and recall information for a short period of time in order to use that information to solve the problem at hand. Table 1 displays some of the difficulties experienced by children with working memory deficits. Researchers in the field of cognitive science have identified three types of memory systems: short-term, working, and long-term memory. Short-term and working memory systems provide temporary storage: short term memory helps to retain information for a few seconds or minutes, while working memory functions as a platform for retrieval of information when it is in immediate use (Hardiman, 2012; Watson & Gable, 2013). In practice (e.g., in psychological evaluations), working memory index scores indicate how well a student performed on tasks requiring him/her to learn and retain information in memory while utilizing the learned information to complete a task. These tasks measure students' skills in attention, concentration, and mental reasoning. The working memory index is closely related to learning and student achievement. Similarly, working memory underpins the academic performance of children with MLD. It is important to draw a distinction between the effects of working memory and short-term memory. Both working memory and short-term memory are related to the achievement of children with MLD; however, the capacity of working memory appears to be a stronger predictor of overall mathematics achievement (Passolunghi et al., 2004; Watson & Gable, 2013).

Mathematics achievement has also been associated with attention and executive functions (EF) (Passolunghi & Siegel, 2004; Bull and Scerif, 2001). Bull and Scerif (2001) even argue that EF (e.g., shifting) is more strongly related to mathematics achievement than reading achievement. However, as pointed out by several researchers, children with MLD have challenges with executive functions (McLean & Hitch, 1999; Passolunghi & Siegel, 2004; Swanson & Sachse-Lee, 2001). Executive functions can be used as predictors of a student's ability to apply different metacognitive strategies. Theoretical views of metacognition emerge from the seminal work of Flavell (1979). Flavell (1979) defines the concept of metacognition as "thinking about thinking." In other words, metacognition is one's own accessible knowledge about one's cognitive processes (Flavell, 1979; Veenman et al., 2006). It refers to the aspect of information processing that monitors, interprets, evaluates, and regulates the contents and processes of its own organization. Flavell's (1979) work laid the foundation for subsequent studies (Geary, 2010; Veenman et al., 2006) and confirmed the importance of student-mediated metacognitive processes as strong predictor for academic success and self-efficacy.

Metacognition is a significant predictor of academic achievement in general, and mathematical performance in particular (Veenman, Van Hout-Wolters, & Afflerbach, 2006). Desoete, Roeyers, and Buysse (2001) conducted two studies to investigate the relationship between metacognition and mathematical problem-solving in 165 students in 3rd grade. The researchers reported that (a) students with MLD had low metacognitive awareness were less efficient in solving mathematical problems, and (b) students with severe MLD showed lower metacognition than students in the moderate and non-MLD groups. The work of Desoete et al. (2001) confirms that students with MLD often experience difficulties with planning mathematical tasks, and demonstrate less efficient, accurate, and appropriate use of mathematical strategies.

Lack of effective instructional practices. Various educators have attempted to remediate MLD by investigating and recommending effective instructional practices for teaching mathematics to K–12 students with MLD. For example, using multiple representations (i.e., concrete, visual, verbal) has been found to foster conceptual understanding of children with MLD (Arcavi, 2003; Booth & Thomas, 1999; Witzel, Mercer, Miller, 2003). While discussing mathematics instruction for students with MLD and those having difficulty learning mathematics, Jayanthi, Gersten, and Baker (2008) urged teachers to incorporate the following instructional practices into their lessons: (a) use explicit instruction on a regular basis; (b) use multiple instructional examples; (c) have students verbalize their reasoning while solving problems; (d) model how to visually represent the information in the problem; (e) model how to solve problems using multiple/heuristic strategies; (f) use formative assessment data to inform instruction; and (g) provide peer-assisted instruction to students.

These practices have been found to improve the mathematical performance of students with MLD. For example, Karp & Voltz (2000) recommend that students with learning disabilities require “instructional models that provide explicit guidance, teacher direction, prompting, and repetition.” (p. 213). Also, student verbalizations (think alouds) of their mathematical reasoning has been shown to increase the mathematical performance of students with MLD (Gersten et al., 2009; Montague, 2008; Naglieri & Johnson, 2000; Rosenzweig, Krawec, & Montague, 2011). Further, verbalizations of mathematical thinking is a metacognitive strategy. Rosenzweig, Krawec, and Montague (2011) studied seventy-three (73) eight graders assigned to three groups: MLD, low achieving in mathematics, and high achieving in mathematics. The researchers verbalized their thinking as they modeled how to solve the problems. All the students were encouraged to verbalize their reasoning as they solved the three problems. Rosenzweig and colleagues (2011) found that verbalizations of mathematical reasoning helped students with MLD internalize metacognitive skills required to solve the word problems.

Several researchers have established the effectiveness of concrete-to-representational-to-abstract (CRA) sequence of instruction. CRA is an effective strategy for improving the mathematics achievement of students with MLD in computation and problem-solving (Arcavi, 2003; Bottage et al., 2007; Ketterlin-Geller et al., 2008; Strickland & Maccini, 2013; Witzel, Mercer, & Miller, 2003). Ketterlin-Geller et al. (2008) found that a graduated instructional sequence that proceeds from concrete to representational to abstract (CRA) benefited struggling students and students with disabilities in elementary and secondary schools. Along similar lines, Strickland and Maccini (2013) used a multiple-probe design approach to investigate the effectiveness of the CRA integration strategy on the ability of three secondary students with MLD. The study results indicate that "integration of the concrete manipulatives, sketches of manipulatives, and abstract notation with the support of a graphic organizer" was an effective strategy to improve the conceptual understanding and procedural fluency of students with MLD.

It has been demonstrated that poorly consolidated number sense contributes to calculation deficits and mathematical difficulties experienced by students with MLD (Gersten et al., 2005). In light of this claim, Woodward (2006) establishes the importance of drill and practice on basic facts for students with MLD. The author conducted an experimental study that involved fifty-eight (58) fourth-graders with a range of academic abilities. Fifteen (15) of the students in the study had IEPs in mathematics. Woodward (2006) found that the integrated approach (i.e., strategies and timed practice drills) and timed practice drills were effective in improving student automaticity in multiplication facts. Students in the integrated approach outperformed the students in the timed practice drills group.

Along these lines, Tournaki (2003) conducted a study that involved forty-two (42) second-grade general education students and forty-two (42) students with learning disabilities (LD). The students were taught basic, one-digit addition facts (e.g., 5 + 3 = __). They received instruction via (a) a minimum addend strategy, (b) drill and practice, or (c) control. The author noted that students with MLD improved significantly only in the strategy condition, as compared to drill-and-practice and control conditions. On the other hand, the general education students improved significantly both in the strategy and the drill-and-practice conditions. Woodward (2006) recommends that drill and practice, as well as explicit strategy instruction, can help students with MLD achieve automaticity in mathematical facts. The study by Owen and Fuchs (2002) corroborates Woodward 's (2006) findings. The performance of students with MLD increased after receiving explicit strategy instruction (Owen & Fuchs, 2002). It is important to note that the study did not focus on the effects of strategy instruction in isolation, they included peer mediation – another intervention for students with MLD (Calhoon & Fuchs, 2003; Maheady, Harper, & Mallette, 2001).

Mnemonic instruction can improve memory and mathematics performance of low-achieving students (Greene, 1999; Kanive, & Ysseldyke, 2013; Manalo, Bunnell, & Stillman, 2000; Nelson, Burns). For example, Greene (1999) investigated the effectiveness of using mnemonics on multiplication fact learning for twenty-three (23) elementary and middle school students with MLD. The students were taught fourteen (14) "difficult-to-memorize" multiplication facts with a combination of mnemonics and traditional instruction. Greene (1999) found that mnemonic instruction supports the learning of students with MLD by helping them retain information or concepts for a long time. The author claims that mnemonic instruction has delivered the greatest learning increases seen in the history of learning disabilities intervention research.

The results of the study conducted by Manalo, Bunnell, and Stillman (2000) are in keeping with Greene (1999) and corroborates the benefits of mnemonic instruction for enhancing number sense and facilitating memory of mathematical facts for students with MLD. Manalo and colleagues describe two types of mnemonics: fact and process mnemonics. Fact mnemonics are used to remember facts and process mnemonics are used to help remember rules, procedures, and principles. In two experiments, Manalo and colleagues examined the effects of process mnemonic (PM) instruction on the computational skills performance of 13- to 14-year-old students with MLD. In the first experiment, twenty-nine (29) students were randomly assigned to four groups: process mnemonic (PM), demonstration-imitation (DI), study skills (SS), or no instruction (NI). In the second experiment, twenty-eight (28) students were assigned to PM, DI, or NI groups. The authors found that the students assigned to the process mnemonic (PM) achieved the highest improvements in addition, subtraction, multiplication, and division.

Factors associated with socioeconomic status. The adverse effect of poverty on learning is well established. In fact, "more students with MLD are found in households living in poverty than in children from the general population." (Cortiella & Horowitz, 2014, p. 19). Also, student subgroups that experience significantly higher rates of poverty could have a higher rate of need for special education and related services. According to National Center for Education Statistics, in the school year 2014-2015, the percentage of children and youth with disabilities receiving services under IDEA for learning disabilities was higher among Hispanic and Black students. A majority of these students are from low-SES households (Cortiella & Horowitz, 2014). In practice, low-SES is often determined by student participation in the free and reduced lunch program.

Students from economically disadvantaged households are likely to struggle in mathematics (Borman & Overman, 2004). This assertion is in keeping with Evans (2004) who reported that children from low-SES families face several environmental barriers that impede their cognitive development and academic progress. Evans (2004) argues that children facing the stressors associated with poverty exhibit lower executive function, slow processing speed, and difficulty with attention. In fact, Jordan, Kaplan, & Hanich (2002) found that children from low-SES families were most at risk for meeting learning disability (in reading and mathematics) criteria by second grade. Bradley & Corwyn (2002) found that children growing up in economically disadvantaged households are more likely to show slower cognitive development than those from mid-SES households. Parents or guardians may not be able to provide enriching experiences (i.e., books, toys, games, and outings) to these children. An annual family income increase of $1,000 was associated with an increase in mathematics test scores of 2.1% of a standard deviation (Dahl & Lochner, 2005). Hackman, Farah, and Meaney (2010) posit that "human brain development occurs within a socioeconomic context and childhood socioeconomic status (SES) influences neural development." (p. 651).

Because most of the students living in economically disadvantaged households are also from culturally and linguistically diverse backgrounds (Cortiella & Horowitz, 2014), the importance of culturally responsive pedagogy and practice cannot be overemphasized (Gay, 2002; Griner, & Stewart, 2013; Howard, 2003). The achievement of students with learning disabilities from culturally and linguistically diverse backgrounds can be enhanced by culturally responsive teaching (Gay, 2002; Howard, 2003). The work of Gay (2002) and Howard (2003) revealed the importance of culturally responsive teaching in special education for ethnically diverse students. Gay (2002) noted that the academic achievement as well as cultural experience, and perspectives of students in both special and regular education could be improved significantly by using culturally responsive instructional practices.

Generally, culturally responsive teaching focuses on practices that include the cultural characteristics, experiences, and perspectives of ethnically diverse students as conduits for teaching them more effectively (Gay, 2002). Further, Gay (2002) asserts that "explicit knowledge about cultural diversity is imperative to meeting the educational needs of ethnically diverse students" (p. 107). Likewise, Banks (2015) stresses that teachers can improve the academic achievement of students from culturally and linguistically diverse backgrounds if they are aware of their cultures, values, language, and learning characteristics. For example, teachers can learn about their students' backgrounds by getting to know their families and attending relevant cultural events. The knowledge acquired by the teachers can be used to develop curriculum and learning activities that reflect the diversity of student body (Nieto, 2008). The awareness of cultural differences present in the classroom would help teachers to understand how diverse cultures of students influence their social behaviors, perceptions, self-esteem, and learning styles.

Conclusion

The first chapter examined (a) the definitions and eligibility criteria for mathematical learning disabilities (MLD), (b) the history of mathematical learning disabilities (MLD), and (c) the factors and underlying causes associated with the achievement gap of students with mathematical learning disabilities (MLD). Although the trends over the last two decades have shown steady progress toward improving the learning outcomes for students with learning disabilities in reading and mathematics, lack of consensus regarding the definition, identification criteria, and classification models of MLD has posed tremendous challenges to the researchers in the field. This chapter also synthesizes research literature to understand factors and underlying causes associated with the achievement gap of students with MLD from cognitive and instructional perspectives. Chapter two examines the factors and underlying causes as they manifest within the author's professional setting (research setting).

CHAPTER 2: A NEEDS ASSESSMENT AT A PUBLIC CHARTER SCHOOL

Introduction

The underachievement of students with MLD enrolled at a public charter school (research setting) in an east coast metropolitan area has been observed for many years. The school currently serves 984 students from nearly every zip code in one of the east coast metropolitan areas. The students at the school represent a broad range of ethnic and socioeconomic backgrounds: Hispanic - 47%, Black - 37%, White - 9%, others - 7%, low income - 73%, special education - 14%, and limited English proficiency - 19%. Soriano (2013) argues that demographic variables can serve as for a proxy from which one can infer some aspects of typical needs, concerns and, community strengths. To this end, sociocultural and demographic profile of the metropolitan area as provided by US Census Bureau include: population estimate in 2013 -- 646,449; educational attainment (bachelor's degree and higher) -- 52.4%; median household income -- $65,830; individuals below poverty level -- 18.6%; White -- 40.1%, Black -- 50.1%, and Hispanic or Latino origin -- 9.6%.

In the metropolitan area where this study is being conducted, only 10% of fourth graders with disabilities reached proficiency on the NAEP test in mathematics in 2015. On the other hand, the percentage of students who performed at or above the NAEP proficient level was 31%. In the research setting, only 26% of students with disabilities achieved proficient or above, and 51% of students without disabilities were at the level of proficient or above in mathematics on the district's standardized assessment in 2014. In spring 2015, students in this metropolitan area took the Partnership for Assessment of Readiness for College and Careers (PARCC) assessments for the first time. The new assessment also confirms the mathematics achievement gap of elementary students with MLD. For instance, on the 2015 PARCC mathematics assessment, no (0%) 4th grader with MLD at the school met the expectations (i.e., achieved proficiency) for grade-level mathematics standards. On the other hand, 17.3% of students without MLD scored at or above proficient on the same math test.

In 2016, only 10% of 4th graders with MLD in the research setting achieved proficiency in mathematics; whereas, 29.4% of students without MLD met or exceeded expectations for grade-level mathematics standards or achieved proficiency in mathematics. None (0%) of the 4th graders with MLD that participated in the 2017 PARCC assessment achieved proficiency in mathematics. It is disheartening that achievement gap between elementary children with and without disabilities widens every year they are in school because schools do not appropriately meet the needs of this student subgroup (Deshler et al., 2001). The mathematics performance of students with MLD at the school (research setting) mirrors the nationwide achievement gap of students with MLD.

Goals and Objectives of the Needs Assessment

Soriano (2013) defines a needs assessment as “a well-thought-out and impartial systematic effort to collect data or information that brings to light or enhances understanding of the need for services and programs” (p.5). The goal of this needs assessment is to determine, in the context of the research setting, the factors and underlying causes associated with the existence and consequences of MLD among fourth graders. Several studies have been conducted on the underlying causes and factors related to the achievement gap of students with MLD. However, the needs assessment specifically examined the factors and underlying causes as they manifest within the setting where the dissertation research is being conducted. The two research questions that guided the needs assessment study are:

- RQ1: Do existing data reliably establish an achievement gap in mathematics performance of students with MLD?

- RQ2: What actionable factors and underlying causes associated with the achievement gap of students with MLD manifest within the research setting?

In order to answer these questions, the researcher (a) collected and analyzed existing data related to the mathematics performance of students with MLD, and (b) conducted a one-on-one interview (semi-structured) with key informants within the organization. The interview included the key components recommended by O'Leary (2014): planning, developing an interview schedule/recording system, piloting and modifying the interview, conducting and analyzing the interview outcomes.

Participants, Selection, and Setting

The research setting comprises of three campuses co-located at the same address: the lower school serves PK-4th grade students, the middle school serves 5th-8th grade students, and the high school serves 9th-12th grade students. These students come from nearly every zip code in the metropolitan area. Extant demographic and academic records data were requested from and provided by the school psychologist and Director of Assessment and Accountability to determine if there is an achievement gap between students with and without MLD. O'Leary (2014) and Soriano (2013) recommend that analyzing existing data and information from key informants' interviews complement each other to provide a combination of qualitative and quantitative data. Therefore, the reviewed data were also used to help determine what information and data were needed from key informants. The researcher collected existing data and interviewed eight key informants who are all highly experienced insiders within the school.

Also, the key informants (i.e., special education teachers, general education teachers, school psychologist, special education coordinator, and the school principal) were they strategically selected to have an array of experience. The interview took place after the participants signed the consent forms (Appendix A) approved by Johns Hopkins University's

Homewood Institutional Review Board (HIRB). The key informant consent form explains the purpose of the study; who is conducting the study; what is expected of the participants; benefits and risks involved; how anonymity and confidentiality will be ensured; and contact information in case of concerns or questions. The interview tool included (a) introduction, (b) key questions, (c) probing questions, (d) closing questions, and (e) summary. “An important advantage of the key informant method lies in the limited number of participants needed, because key informants are presumed to have a broad knowledge of needs within the targeted area” (Soriano, 2013, p. 125). One-to-one interviews with key informants was preferred and used instead of focus group interviews in order to get candid and in-depth qualitative data, as well as ensure confidentiality. The focus group dynamic can also prevent some participants from voicing their opinions about sensitive topics.

Planning the key informant interview involved (a) identifying key informants, (b) notifying and getting consent from key informants, (c) choosing type of interview, (d) developing an interview tool, (e) determining documentation method, (f) conducting key informant interviews, and (g) compiling, organizing, and analyzing key informant interview data. Demographic characteristics of participants/key informants are presented in Appendix B. The researcher scheduled not more than two interviews per day. After each interview, the researcher took some time to make additional notes and organize initial findings and impressions. Although the interview was about thirty minutes, several informants were willing to speak longer. As recommended (O’Leary, 2014), at sometimes during the interviews, the researcher deviated from the plan to pursue interesting tangents or to follow up on unexpected data/points that emerged. To ensure descriptive validity, the researcher recorded the interview responses in three ways: (a) notes written during the interview; (b) notes written afterwards; and (c) audio recordings of the interview responses. A follow-up appreciation note was sent to each participant after the interview.

Instrumentation, Measures, and Variables

The following key constructs and concepts associated with the dissertation research are addressed in this chapter: (a) mathematical learning disabilities (MLD), (b) achievement gap, (c) socioeconomic status (SES), (d) factors and underlying causes associated with the achievement gap, and (e) instructional practices for students with MLD. It is important to note that the ordinal variable, socioeconomic status (SES) is included in the section because all the students participating in this study are from low-SES households.

Mathematical learning disabilities (MLD). The American Psychiatric Association defines MLD as a discrepancy between performance on mathematics achievement tests and expected performance based on age, intelligence, and years of education. Nationwide, approximately 7% of students have MLD, and another 10% show persistent low achievement (LA) in mathematics despite competence in most other areas (Shalev, Manor, & Gross-Tsur, 2005; Geary, 2011). In light of the inconsistencies in definitions and criteria (practice and research) used in classifying students as having MLD (Mazzocco, 2007; Watson & Gable, 2013), this present study adopts the federal definition and guidelines. A student with MLD is operationally defined as a child with an Individual Educational Program (IEP) with a designation of Specific Learning Disability (SLD) in mathematics. Based on this definition, only 7% of students enrolled at research setting met the criteria for learning disabilities in mathematics. The metropolitan area where the research setting is located defines SLD as “a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations.” Federal and State regulation reference: 34 CFR

§300.8(c)(10), 5 E DCMR 3001.1. There are two MLD identification models proposed by the school district: discrepancy model and the scientific research-based interventions model (Response to Intervention – RTI).

The research setting implements the discrepancy model to make MLD (SLD in mathematics) eligibility determination. This model involves three main criteria: (a) the student does not achieve adequately and/or does not make sufficient progress to meet age or state-approved grade-level standards in mathematics, reading and/or written expression, when provided with learning experiences and instruction appropriate for the student's age or state-approved grade-level standards; (b) the student must demonstrate a discrepancy between achievement (as measured by the academic evaluation) and measured ability (as measured by the intellectual evaluation) of two years below a student's chronological age and/or at least two standard deviations below the student's cognitive ability as measured by appropriate standardized diagnostic instruments and procedures; and (c) student's underachievement must not be related to visual, hearing, or motor disabilities; intellectual disability (known as mental retardation); emotional disturbance; cultural, factors; environmental or economic disadvantage; or limited English proficiency. Figure 2 shows the SLD eligibility criteria under the discrepancy model. Only four fourth-graders with MLD are included in this dissertation research due to these eligibility criteria.

Option A: Discrepancy Model	
Criterion 1: The student does not achieve adequately and/or does not make sufficient progress to meet age or State-approved grade-level standards in one or more of the following areas, when provided with learning experiences and instruction appropriate for the student's age or State-approved grade-level standards (At least one of the following must be marked in order to meet the requirement): ☐ Oral expression ☐ Reading fluency skills ☐ Listening comprehension ☐ Reading comprehension ☐ Written expression ☐ Mathematics calculation ☐ Basic reading skill ☐ Mathematics problem solving Basis for determination: ____________________ ____________________ ____________________	
___ Yes ___ No	**Criterion 2: The student demonstrates a discrepancy between achievement (as measured by the academic evaluation) and measured ability (as measured by the intellectual evaluation) of two years below a student's chronological age and/or at least two standard deviations below the student's cognitive ability as measured by appropriate standardized diagnostic instruments and procedures.** (Must be yes in order to meet the requirement)
Criterion 3: Is the impact on the student's achievement level the result of: (All of the following must be no in order to meet the requirement)	
___ Yes ___ No	Lack of appropriate instruction in reading, to include the essential components of reading instruction (phonemic awareness, phonics, fluency, vocabulary and comprehension)
___ Yes ___ No	Lack of appropriate instruction in math
___ Yes ___ No	Lack of appropriate instruction in writing
___ Yes ___ No	A visual, hearing, or motor disability
___ Yes ___ No	Intellectual disability (known as mental retardation)
___ Yes ___ No	Emotional disturbance
___ Yes ___ No	Cultural factors
___ Yes ___ No	Environmental or economic disadvantage
___ Yes ___ No	Limited English Proficiency

Figure 2. SLD Discrepancy model with eligibility criteria.

Many researchers and practitioners have criticized the discrepancy model (Mazzocco, 2007). The prominent argument against the model is the lack of reliable and sufficient data to establish a discrepancy until a child is in third and fourth grade. It is difficult to find a discrepancy before third or fourth grade. Therefore, the critics label the IQ-achievement discrepancy model/approach as a "wait-to-fail" model (Mazzocco, 2007).

Achievement gap. According to NAEP (also known as The Nations Report Card), an achievement gap occurs when one group of students significantly outperforms another group. Along similar lines, the present study operationally defines achievement gap as the difference between the standardized test scores of students with MLD and students without MLD. The needs assessment aimed at determining the extent of the achievement gap of students with MLD within the organization using the District's standardized test and Partnership for Assessment of Readiness for College and Careers (PARCC). It is important to note that 2013-2014 was the final school year in which the District's standardized assessment was administered. The district adopted the PARCC assessments for Mathematics and English Language Arts (including Composition) during the 2014-2015 session.

Socioeconomic status (SES). Oakes and Rossi (2003) define socioeconomic status (SES) as “a construct that reflects one’s access to collectively desired resources, be they material goods, money, power, friendship networks, healthcare, leisure time, or educational opportunities.” (p. 5). Other researchers have argued that SES is difficult to define and measure. For example, Guo and Harris (2000) posit the view that SES status is difficult to measure directly, and there is no consensus on its best measures. However, well-known univariate or proxy measures of SES include income, wealth, educational attainment, poverty, and residential neighborhood (Oakes & Rossi, 2003; Sirin, 2005).

In the research setting, eligibility for free or reduced-price meals under the Department of Agriculture's National School Lunch Program (NSLP) is used as a measure for low-SES. Based on these criteria, seventy-three percent (73%) of the students are from low-SES households. Also, all the students that participated in this study are from low-SES households -- they participate in the free or reduced-price meals program. As discussed in chapter one, students from low-SES household are more likely to struggle in mathematics than those from mid/high-SES households (Guo & Harris, 2000). Guo and Harris (2000) noted that mathematics difficulties among low-SES students are related to cognitive impairments brought about by the stressors of poverty. Relatedly, Sirin (2005) reviewed several journal articles (between 1990 and 2000) on socioeconomic status (SES) and academic achievement and reported that poverty accounts for up to 60% of the variance in standardized test scores.

Data Collection and Analysis

Existing data. "What better way to meet information needs than to use existing data, also called secondary data" (Soriano, 2013, p. 76). Secondary data were collected and analyzed (descriptive statistics) to determine if an achievement gap exists between students with and without MLD at the school. O'Leary's (2014) basic steps of secondary data analysis informed the process of determining research questions, locating data, evaluating relevance of the data, assessing credibility of the data, and conducting analyses. The school's Director of Assessment and Accountability provided extant data (test scores) from 2009-2014. After the raw data were collected, means were calculated (sum of all the data ÷ sample size) to determine achievement gaps between student subgroups and in different years (See figure 1). Soriano (2013) warns that existing data "seldom contain all of the needed information." (p. 76). Considering this recommendation, the researcher collected and reviewed existing research data before determining what additional information needs to be collected from key informants within the organization.

Key informants interview. The researcher conducted face-to-face interviews with two General Education teachers, two Special Education Teachers, Special Education Coordinator, School Psychologist, Director of Assessments and Accountability, and School Principal. This is to ensure that the needs assessment results reflect varying perspectives. One-to-one interviews with key informants were preferred and used because a focus group dynamic can prevent some participants from voicing their opinions about topics or giving in-depth answers. The written protocol and consent forms were given to the needs assessment study participants. The key informant consent form explains the purpose of the study; who is conducting the study; what is expected of the participants; benefits and risks involved; how anonymity and confidentiality will be ensured; and contact information in case of concerns or questions.

The main components of the interview tool are introduction, key questions, probing questions, closing questions, and summary. The researcher/interviewer scheduled not more than two interviews (semi-structured) per day. After each interview, the interviewer took some time to make additional notes and organize initial findings or impressions. Although the interview was thirty minutes, several informants were willing to speak longer. Few times during the interviews, the research deviated from the plan to pursue interesting tangents or unexpected data that emerged (O'Leary, 2014). In order to ensure descriptive validity, the researcher recorded the interview responses in three ways: notes written during the interview, notes written afterwards, and audio recording the interview responses. A follow-up "thank you" note was sent to each participant after the interview.

Qualitative Data Analysis (QDA). O'Leary's (2014) process of reflective qualitative analysis informed the interview analysis. The researcher (a) collected and organized raw data; (b) coded and categorized data; (c) searched for meaning through thematic analysis; (d) interpreted meaning; and (e) drew conclusions. O'Leary (2014) also recommend the use of QDA software for effective and efficient handling of a large data set and manual handling for small data set. The researcher used manual QDA due to the manageable amount of key informants' interview data. The manual approach gave more control over and ownership of the work (Saldana, 2009). A major part of the QDA was coding. Saldana (2009) defines it as "a word or short phrase that symbolically assigns a summative, salient, essence-capturing, and/or evocative attribute for a portion of language-based or visual data" (p. 3). In additional to preliminary jottings (Saldana, 2009), the researcher used codes based on keywords and phrases found in literature relevant to dissertation study as well as direct quote/phrases (In Vivo Code) from the interviewees. See Appendix C for codes and categories.

Challenges associated with the key informant interviews. According to O'Leary (2014), six challenges associated with key informant interviews are: (a) identifying an informant with characteristics that match research focus and process; (b) confirming the status of those identified – do they really have the experience and insider knowledge that will inform the needs assessment in a credible way? (c) gathering open and honest information from key informants; (d) recognizing informant subjectivity (e) managing bias and ethical issues. A notable challenge faced was the impact of note taking on the flow of interview conversations, however, audio recording was very helpful in this area.

Needs Assessment Results

Figure 3 depicts the percentage of students with and without disabilities that performed on or above grade level between 2009 and 2014. Similarly, Table 2 shows the achievement of students with and without MLD on the PARCC tests in mathematics between 2015 and 2017. Both standardized assessment results (from 2009-2017) confirm the achievement gap of students with MLD.

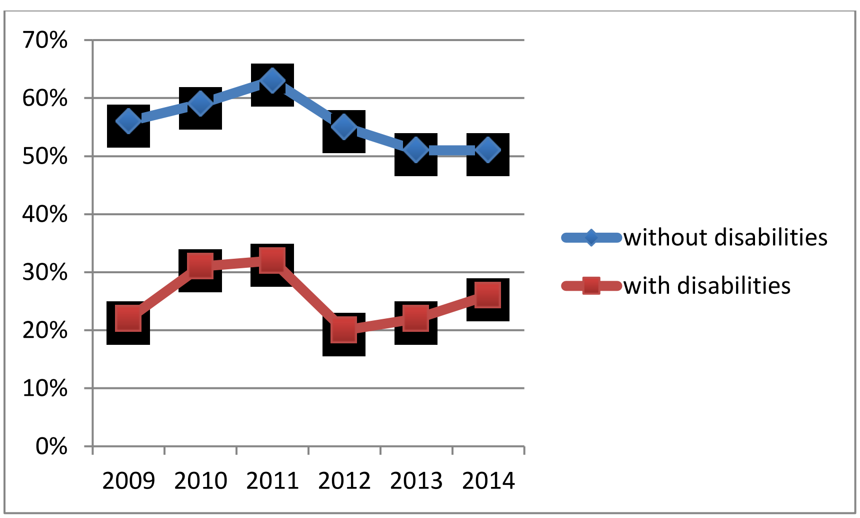


Figure 3. The percentage of students with and without disabilities classified as proficient on the District's mathematics tests between 2009 and 2014.

Table 2

Performance of students with and without MLD on the PARCC mathematics test between 2015 and 2017.

	Students with MLD	Students without MLD
2015	12.5%	17.8%
2016	28.8%	26.7%
2017	0.0%	29.0%

Note. The table displays the percentage of elementary students that met and exceeded PARCC grade level expectations in mathematics between 2015 and 2017.

Factors and underlying causes associated with the underachievement of students with MLD within the school. The key informants' interviews generated important factors and underlying causes associated with the achievement gap of children with MLD in the research setting. These underlying causes and factors include:

- Weak Cognitive Abilities: The school psychologist and special education teachers remarked that verbal comprehension, perceptual reasoning, weak visual perceptual skills, weak working memory, and slow processing speed are some of the cognitive processes that underlie the academic struggles and underachievement of children with MLD. The students with MLD exhibit difficulties in procedural fluency and conceptual understanding due to underdeveloped cognitive mechanisms. In the classroom, students with MLD demonstrate behaviors such as (a) difficulty processing information; (b) difficulty identifying the relevant information and facts in mathematics problems; (c) difficulty maintaining attention; and (d) difficulty with self-monitoring and self-regulation during problem-solving; impulsive.

- Inappropriate instruction or lack of evidence-based practices: The special education coordinator and special education teachers reported that the teachers do not consistently implement strategies and practices that support the learning needs of students with MLD. For example, lack of hands-on activities to build conceptual understanding, lack of multisensory approaches, insufficient time to practice the mathematical skills; inappropriate pace of instruction for students with disabilities, insufficient teacher feedback; overuse of constructivist approach; underuse of explicit instruction and teacher modeling, and limited opportunities for students to verbalize (think aloud) their mathematical thinking were some of the inadequacies mentioned by the interviewees.
- Socioeconomic factors / home factors: The low-SES status of parents, inability of parents to pay for tutors and summer programs, parent education level, minimal parental involvement, lack of self-esteem, and parents not knowing how to advocate for children may also be related to the underachievement of children with MLD from low-SES household.
- School factors: There are organizational factors that might have contributed to the achievement gap of children with MLD. For example, lack of teacher accountability; paucity of research-based mathematics interventions (Tier II and Tier III); IEPs are not individualized (i.e., not properly addressing the child's needs); lack of teacher training in Common Core State Standards; the rigor (depth and breadth) of Common Core State Standards; complexity of mathematics; low expectations for students with disabilities; improper implementation of different co-teaching models; and lack of culturally responsive practices among teachers may be related to the achievement gap of students with MLD.

Conclusion

Seven percent (7%) of students enrolled at the school, consistent with the prevalence of MLD in school-age population across the country (Geary, 2011), have learning disabilities in mathematics and are required to receive Free Appropriate Public Education (FAPE). Despite this promise, existing interventions and policies have not been able to drastically reduce the achievement gap of students with MLD. Most schools fail to make AYP most often because of the students with disabilities subgroup (Eckes & Swando, 2009). In the context of the research setting, the needs assessment study confirms an achievement gap in mathematics. The key informants revealed many factors that might have contributed to the achievement gap of students with MLD at the school. The information gathered from the needs assessments, especially the those that are related to instructional practices informed the focus of the intervention and dissertation study. The limitations of the needs assessment study require cautious interpretation of the outcomes. For example, the study only collected standardized assessment data (e.g., PARCC). Data and information from the IEP progress reports, formative assessments, and summative assessments were not considered.

CHAPTER 3: A REVIEW OF LITERATURE (INTERVENTION)

The needs assessment study revealed that inappropriate instruction and lack of evidence-based practices and interventions might be associated with the underachievement of students with MLD in the school. Targeted interventions and best practices aimed at remediating the mathematical skill deficits of children with MLD may involve several components such as (a) direct, explicit instruction; (b) student verbalizations of their mathematical thinking; (c) use of visual representations while solving mathematical problems; (d) providing opportunities for repeated practice; and (e) providing timely and corrective feedback (Furlong, McLoughlin, McGilloway, & Geary, 2016; Gersten et al., 2009; Gersten & Clarke, 2007; Kroesbergen 2003). This study focuses on student verbalizations (student think-alouds) due to the strategy's large effect size for special education students (Gersten & Clarke, 2007; Gersten et al., 2009).

In research studies that investigated mathematical problem-solving skills for students with MLD, verbalization is often defined as the act of verbally stating one's thinking processes (Montague, 2008; Naglieri & Johnson, 2000; Rosenzweig, Krawec, & Montague, 2011). Student expression also include expressing one's mathematical reasoning through writing and drawing (Baker, Gersten, & Lee 2002; Gersten et al. 2008). A considerable amount of literature has been published on how different think-aloud/verbalization approaches are used in special education: (a) students can verbalize the specific steps that lead to the solution of the word problem (problem-specific approach) or generic heuristic steps (generic approach) that are related to the mathematics problems (Jayanthi, Gersten, & Baker, 2008); (b) students can verbalize their mathematical thinking by asking themselves questions (Rosenzweig, Krawec, & Montague, 2011) and; (c) students can verbalize their reasoning and steps in a solution format (Tournaki, 2003).

Researchers in the field of mathematics interventions for students with MLD suggest that teachers should encourage students with MLD to verbalize their mathematical thinking using specific strategies (Hutchinson, 1993; Montague, 2008; Rosenzweig, Krawec, & Montague, 2011). These strategies and steps include (a) explicitly modeling the think-aloud strategy while solving mathematical problems, (b) providing students with a set of questions, prompts, or templates for thinking aloud, (c) encouraging students to select an appropriate representation, (d) guided practice, (e) corrective feedback, and (f) frequent cumulative review (Hutchinson, 1993; Montague, 2008; Naglieri & Johnson, 2000; Rosenzweig, Krawec, & Montague, 2011).

Student Verbalization as an Evidence-Based Strategy

Experts and researchers have recommended specific quality indicators that can be used to determine whether a practice or intervention may be considered evidence-based. For example, Gersten and colleagues (2005) recommended quality indicators for experimental and quasi-experimental designs in special education. These quality indicators include assignment of participants, mortality equivalence, no ceiling or floor effects, pretest equivalence, extensive instructor training, Hawthorne effect controlled, treatment fidelity, type of control condition, and teacher effects controlled. Similarly, Horner, Carr, Halle, McGee, Odom, and Wolery (2005) identified seven quality indicators that must be present in single-subject research design to support evidence-based practice in the context of special education. These quality indicators are: (a) participant and setting descriptions; (b) independent variables; (c) dependent variables; (d) baseline measurement; (e) experimental control, or internal validity; (f) external validity; and (g) social validity. Further, Horner and colleagues (2005) proposed that an intervention or instructional practice could be considered an evidence-based practice based on a minimum of five well-designed single-case studies, conducted by at least three independent researchers with a total of twenty or more subjects across studies.

Several randomized control trials, quasi-experimental, and single-case experimental design studies that included these quality indicators have established verbalization of mathematical thinking (think aloud) as an evidence-based intervention and strategy for teaching students with difficulties in mathematics (Montague, 2008; Naglieri & Johnson, 2000; Rosenzweig et al., 2011). Also, several meta-analyses conducted on studies that investigated evidence-based practices for teaching students with difficulties and disabilities in mathematics have reported a large effect size for student verbalizations (Baker et al., 2002; Gersten & Clarke, 2007; Gersten et al., 2009; Kroesbergen & Van Luit, 2003). For example, Gersten and colleagues (2009) proposed that the weighted effect size of group experimental studies must be greater than zero before a practice could be considered an evidence-based. In this regard, research studies that focus on mathematics interventions for students with MLD and other low-achieving students revealed that the mean effect size for student verbalization (think-aloud) is 1.04 (range of 0.07 to 2.01). This mean effect size implies that the verbalizations might raise students' scores on a standardized test about 25 percentile points (Baker, Gersten, & Lee 2002; Gersten et al. 2008).

Search Protocol

A systematic literature search (online/computerized search) using education research databases such as ERIC, Education Full Text, PsycINFO, and education journals was conducted. The search was completed in order to (a) gain a thorough understanding of the student verbalizations as an evidence-based strategy/intervention; (b) identify and understand seminal works, potential areas for research and studies related to the intervention; (c) compare and critique existing findings; and (d) identify and establish methodological focus and theoretical framework for the use of think-aloud as an intervention. The scope of this study and search protocol (search terms and criteria for inclusion) include studies that focus on (a) student verbalization or think-aloud, (b) mathematical problem solving, (c) elementary and secondary school students with mathematical learning disabilities, (d) peer-reviewed journal publications between 1985-2015 (in order to include seminal work), and (e) experimental design with a control or comparison group or a single-case experimental design. These search procedures for the period between 1985 and 2015 resulted in the identification of 47 studies. Of this total, 26 studies were selected for further review based on the review and analysis of the abstracts and research design. Of these 26 studies that met the criteria for inclusion in the use of think-aloud as an intervention study, only 10 studies were selected based on the quality indicator recommendations from Gersten and colleagues (2005).

The articles were read, and studies that met the above criteria were retained. The studies that used a group design with random assignment to intervention and comparison groups are currently considered the “gold standard” in special education research (Gersten et al., 2005). The studies included in this assignment are quality-group studies with randomized controlled trial (RCT). In order to ensure methodological quality (Gersten et al., 2005), the studies include detailed descriptions of participants, setting, fidelity of implementation data, independent variable, and services provided in the comparison group. It is also important to state that these studies have weighted effect size that is greater than zero (Gersten et al., 2005).

Evaluation of Research Quality (Literature Findings)

Student verbalization or think-aloud has been identified as an evidence-based practice that can be used to teach cognitive and metacognitive strategies to students with MLD (Baker, Gersten, & Lee 2002; Gersten et al. 2008; Naglieri & Johnson, 2000; National Mathematics Advisory Panel, 2008; Rosenzweiz, Krawec, & Montague, 2011; Tournaki, 2003). Verbalization is the act of orally stating one’s thinking processes or reasoning by talking to peers, teacher, or self. The strategy also includes writing or drawing the steps that the students employed in solving the math problems. The research studies synthesized below show how teacher verbalizations or think aloud, used as part of explicit or direct instruction, contributed to math performance among students with MLD.

Schunk and Cox (1986) randomly assigned ninety middle school students with MLD to three groups in order to investigate the impact of continuous student verbalizations versus no student verbalizations. The researchers provided the same explicit instruction of a subtraction skill during the six sessions of forty-five minute interventions. The first group was asked to think aloud while working on the task; the second group started verbalizing, but eventually stopped; and the third group did not verbalize their reasoning while solving the math computations. The researchers reported a direct correlation between explicit teacher instruction, student verbalizations, and improved student practice. Additionally, Schunk and Cox (1986) found that (a) verbalizing each step of the computation process helped students with MLD remain engaged; (b) student verbalizations supported cognitive strategy acquisition; and (c) student verbalizations helped students with MLD retain and recall important information. One of the limitations of this study is that the researcher did not enhance student verbalization through teacher questioning or a template for think aloud, resulting in a small effect size (0.07).

The largest effect size ever reported for think aloud or verbalization was 2.01 in a study by Marzola (1987). The researcher investigated the instructional approach for teaching children with learning disabilities how to solve arithmetic word problems. The study involved sixty 5th and 6th grade students with learning disabilities. The students in the experimental group were taught how to problem solve steps using metacognitive strategies (e.g., verbalization). Both the experimental group and the control group were allowed to use calculators during the twelve sessions of thirty minutes. The researcher found that students with learning disabilities in the experimental group performed better than those in the control group. Marzola's (1987) results must be seen in light of its limitations: the control group received no instruction at all, just feedback on the accuracy of their work. This approach might have influenced the effect size.

Hutchinson (1993) conducted a study involving twenty students with MLD in grades eight thought ten. These students were randomly assigned to either the treatment group (12 students) or control group (8 students). The students in the treatment group received explicit instruction on the use of specific strategies for solving word problems during the sixty intervention sessions of forty minutes. The intervention for the treatment group was comprised of (a) modeling and verbalizing cognitive and metacognitive strategies; (b) providing feedback during guided practice sessions; and (c) providing a prompt card for self-questioning to students. On the other hand, the students with MLD assigned to the control group did not receive the cognitive and metacognitive strategy instruction, but they were asked to verbalize their reasoning while solving the same word problems. Hutchinson (1993) found that the students with MLD in the treatment group performed better those in the control group. Also, students in the treatment group recalled almost all the steps that the teachers had used while modeling the strategies. This corroborates prior findings described by Schunk and Cox (1986) that students with MLD can effectively retain and recall math strategies when they are given opportunities to verbalize their mathematical thinking. It is also important to note that students with MLD in the treatment group recognized their errors and self-corrected (a metacognitive strategy) while solving the word problems. This might be related to the fact that Hutchinson (1993) encouraged students to ask themselves relevant questions. For example, “Have I written an equation?” or “Have I expanded the terms?” (p. 39).

In order to examine the types of errors common among students with MLD, Parmar (1992) conducted an investigation that involved thirty-one middle school students with MLD. The students were given word problems and asked to verbalize their reasoning. The teacher collected and transcribed the notes on student verbalizations. The researcher found that students' errors were associated with skill and knowledge deficits. These deficits can be addressed when teachers model and verbalize the use of cognitive and metacognitive strategies while solving word problems (Montague, 2003). Montague (2003) further noted that cognitive and metacognitive strategies are more effective when used together than when used alone.

In order to facilitate the integration of the cognitive and metacognitive strategies, Montague, Warger, and Morgan (2000) discuss and support the implementation of "Solve It!," a mathematical framework for students who have difficulty solving mathematical word problems. The Solve it! framework has been shown to foster the use of cognitive processes and self-regulation strategies among students with MLD. Solve it! involves seven cognitive activities -- reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking. The framework had helped students with MLD and other struggling learners internalize cognitive strategies, especially when the students verbalized their reasoning while solving word problems (Montague, 2003). The cognitive activities become automatic during problem-solving through practice.

Cognitive planning can be a major hurdle for students with MLD trying to utilize cognitive strategies (Naglieri and Johnson, 2000). To investigate whether students with MLD have cognitive planning weaknesses and could benefit from cognitive strategy instruction with specific attention to planning Nagliere and Johnson conducted an intervention study. Their study involved nineteen middle school students with MLD randomly assigned to five groups (four groups with a cognitive weakness and one group with no cognitive weakness). The students received instruction in the cognitive problem-solving strategy, PASS (Planning, Attention, Simultaneous, Successive). Students verbalized their reasoning as they implemented the PASS strategy, and teachers used prompts and questions to probe student reasoning if no verbalizations were heard after five seconds. Naglieri and Johnson (2000) reported that students who demonstrated weaknesses in planning benefited the most from the verbalization process with an effect size of 1.4. This result emphasizes the need for teachers to encourage students with MLD to verbalize their mathematical reasoning as they use different cognitive strategies.

Rosenzweig, Krawec, and Montague (2011) have identified metacognition as an area of deficiency among students with MLD. Their study involved seventy-three students in eight-grade assigned to three groups: MLD, low achieving in mathematics, and high achieving in mathematics. The researchers verbalized their thinking as they used metacognitive skills while solving math problems. All students were encouraged to think aloud and practice the strategies while they solved three math problems. The verbalizations were recorded, transcribed, and analyzed to know whether they were metacognitive or not. The authors found that students with MLD experienced frustrations as the word problems increased in complexity. This resulted in more nonproductive metacognitive behaviors (e.g., emotional reactions, negative self-talk) among these students. Rosenzweig et al. (2011) recommended that students with MLD receive explicit instruction on how to use metacognitive strategies while solving math problems. The study of Rosenzweig and colleagues (2011) established that verbalization strategies could help students with MLD internalize metacognitive skills required to solve word problems.

Benefits of Student Verbalizations

Several advantages of student verbalizations have been reported in research literature: (a) it helps students with MLD to clarify what they do and do not understand (Bosson et al., 2010; Parmar, 1992; Tournaki, 2003); (b) it helps teachers to monitor and support the progress of students' learning; (c) verbalizing steps in problem-solving may address students’ impulsivity as well as facilitate self-regulation during problem-solving (National Mathematics Advisory Panel, 2008); (d) students with MLD benefit from the strategy more than other student subgroups included in the studies reported (Hutchinson, 1993; Naglieri & Johnson, 2000); (e) student verbalization or think-aloud strategy is related to improved mathematics test scores (Swanson, 1990); (f) the strategy or practice helps teachers to determine the types of cognitive and metacognitive strategies students used and errors made while solving the problem (Parmar, 1992; Rosenzweig, Krawec, & Montague, 2011); and (g) verbalizing each step of the process helps students with MLD recall relevant information and also support strategy (e.g., mnemonics) acquisition (Schunk & Cox, 1986).

According to the report by the National Mathematics Advisory Panel (2008), an effective instructional approach for students with learning disabilities should be explicit and systematic. Explicit instruction involves providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review. Student verbalization is often preceded by an explicit instruction on a cognitive strategy that requires teachers to think-aloud (Hutchinson, 1993; Rosenzweig, Krawec, & Montague, 2011; Schunk & Cox, 1986). Furthermore, Schunk and Cox (1986) noted that there is a direct relationship between explicit instruction, student verbalizations, and student mathematics achievement. In this regard, teachers of students with MLD should know how to verbalize their reasoning while solving word problems. Similarly, Baker et al. (2002) and Gersten et al. (2008) urge educators to provide clear expectations about the strategy. For example, teachers must let the students know that “thinking aloud” is an important part of the task. When students know that it is required, then there is more likely to be a conscious effort to use the strategy as modeled by the teachers. Teachers need to model the process of thinking aloud by explicitly stating what it means to “explain your thinking.”

Several researchers and experts have provided clear guidelines on how to implement verbalization strategies in the classroom. For example, educators of students with learning disabilities are urged to (a) specify the steps in the cognitive and metacognitive strategies (Rosenzweig, Krawec, & Montague, 2011); (b) model the steps in the strategy by verbalizing thoughts (thinking aloud) as each step is completed (Hutchinson, 1993; Naglieri & Johnson, 2000; Rosenzweig, Krawec, & Montague, 2011; Schunk & Cox, 1986); (c) provide guided practice -- teachers use probes and prompts to enhance student verbalizations (Naglieri & Johnson, 2000); (d) allow the students to implement the strategy (apply the steps) and verbalize their reasoning as they independently complete the word problems (Hutchinson, 1993; Naglieri & Johnson, 2000; Rosenzweig, Krawec, & Montague, 2011; Schunk and Cox, 1986); and (e) provide corrective feedback to students and fade support as the students master the strategy (Hutchinson, 1993; Schunk & Cox, 1986; Tournaki, 2003).

Furthermore, a synthesis of the research studies revealed that teachers can promote verbalizations by (a) giving their students a series of prompts -- questions or sentence starters -- to guide them through the process of thinking aloud, especially questions that require them to justify math decisions; (b) allowing students with MLD to use pictures or diagrams to support or explain their thinking; and (c) inviting a peer to listen and comment on the content while the teacher concentrate on the student's use of the strategy. Also, a small group activity might work better for some students because it offers them a chance to hear others before sharing their thinking. The summaries of studies are presented in Table 3.

Research Quality Coding

Ten studies were coded on the following variables proposed by Gersten et al. (2005): research design, quantity of research, methodological quality, and magnitude of effect. "Likert-type scales have successfully been applied to quality indicators measuring the evidence base of research" (Nagro & Cornelius, 2013, p. 316). A 3-point Likert-type scale was used to evaluate the studies included in this review. The scale ranges from 0 to 2: a score of 0 was assigned when the quality indicator was not met or not reported on; a score of 1 was assigned when the quality indicator was somewhat met; and a score of 2 was assigned when the quality indicator was completely met. For example, because weighted effect size of group experimental studies must be greater than zero (Gersten et al., 2005), the researcher assigned 2 to studies with an effect size greater than zero, 1 to studies with no (zero) effect size, and 0 to studies with an effect size that was less than 0 (negative).

Determinations of Evidence Base

This section includes (a) summary of the findings using two figures to display the overall research quality for the body of research, (b) determinations of evidence base, (c) limitations of the review, and (d) recommendations for practical use and future research. Figures 4 and 5 show the overall research quality for the body of research included in this chapter. Table 3 displays the quality indicators for research studies by domain. Table 4 shows the summary of studies.

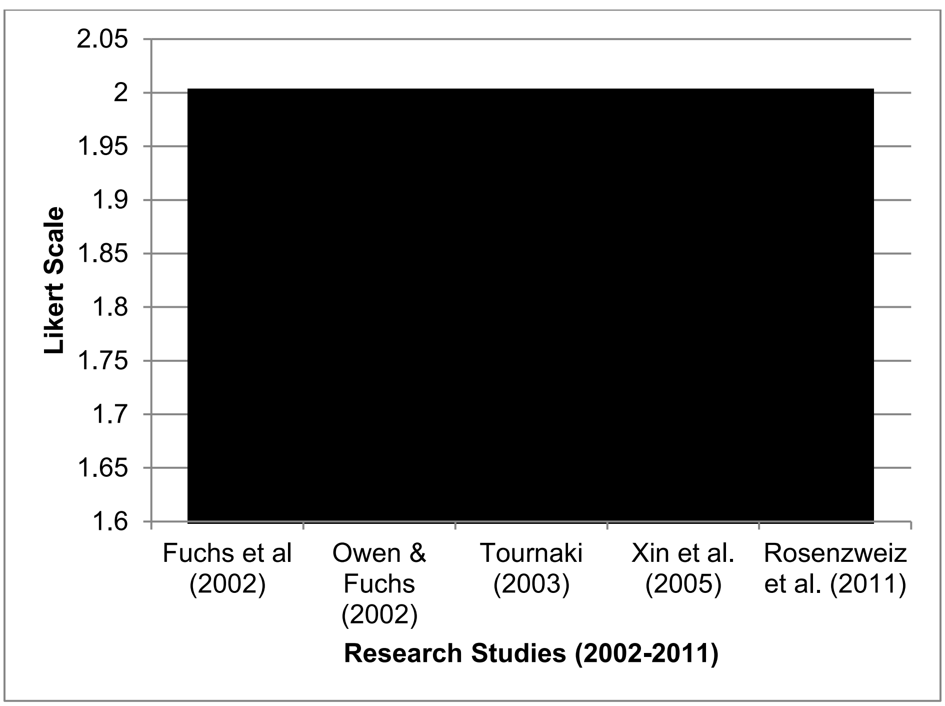


Figure 4. Research quality for studies between 2002 and 2011.

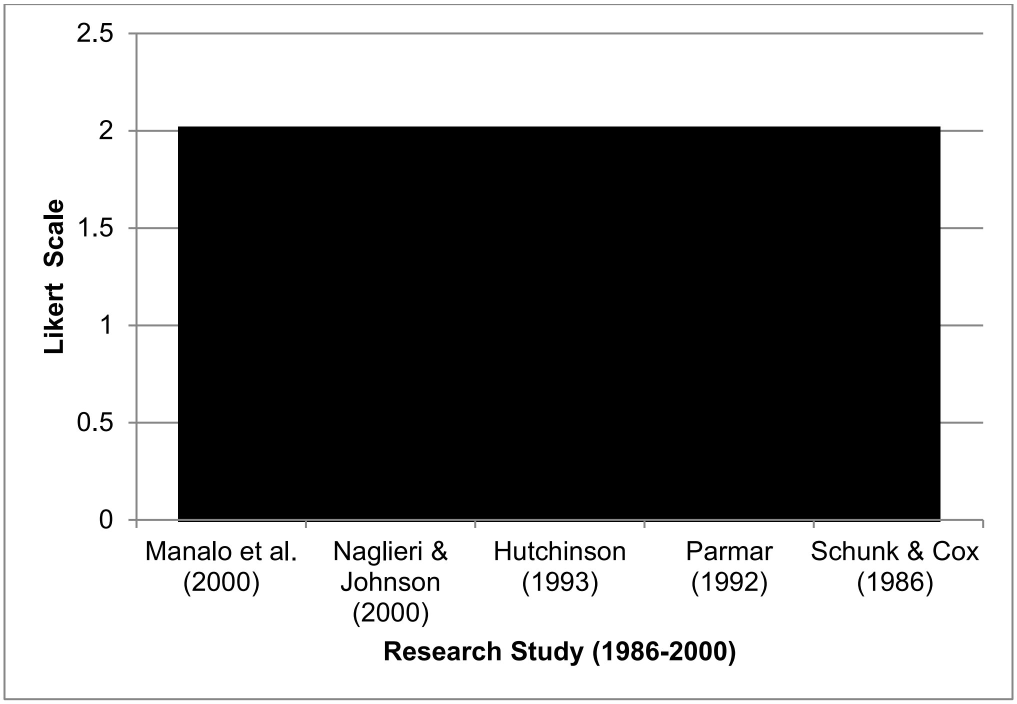


Figure 5. Research quality for studies between 1986 and 2000.

Gersten et al. (2005) noted that an intervention or practice could be identified as evidence-based when "there are at least four acceptable quality studies or two high qualities that support the practice, and the weighted effect size is significantly greater than zero." (p. 162). The studies reviewed in this assignment are considered "high quality" based on the criteria established by Nagro & Cornelius (2013). These studies establish student verbalization as an evidence-based intervention or practice for teaching mathematics to students with MLD. Also, the participant selection criteria and characteristics support the generalizability of the findings (Gersten et al., 2005). In order to ascertain an evidence-based practice, multiple replications across participants and researchers are required (Horner et al., 2005). Further, Gersten et al. (2004) propose that random assignment of participants to experimental groups is one indicator of high-quality group design research. All the studies in this assignment met this requirement. Lastly, the mean effect size for student verbalization is 1.04.

Limitations of Literature Review

The conclusions of the present review must be seen in light of their limitations. The conclusions require cautious interpretation as several of the studies reported short duration of the intervention and recorded no fidelity of implementation. Although research to date has not offered specific guidelines as to the acceptable levels or standards of implementation fidelity (Smith, Daunic, & Taylor, 2007), implementation fidelity is a critical feature of any intervention (Gersten et al., 2005). Tournaki (2003), Hutchinson (1993), Fuchs et al. (2002), Manalo, Bunnell, and Stillman's (2000) studies are somewhat weakened by their lack of data on treatment fidelity. These researchers failed to document the methodological strategies used to monitor and enhance the reliability and validity of their interventions. Additionally, the findings and recommendations discussed in this chapter should be tempered somewhat because of the variations in the duration of the intervention. For example, Hutchinson (1993) implemented the student think-aloud intervention for sixty sessions of forty minutes, whereas Tournaki (2003) implemented the same intervention for eight sessions of fifteen minutes, but they both reported similar effect sizes.

Recommendations for Practical Use and Future Research

In order to implement and foster student verbalization strategies in the classroom, teachers of students with MLD need to (a) emphasize the importance and benefits of explaining one's reasoning to the students; (b) model how to verbalize mathematical reasoning while solving mathematical problems; (c) effectively model the use of cognitive and metacognitive strategies; (d) use prompts, sentence starters, and ask guiding questions to help students with MLD focus on their reasoning, not just the solution, even when their answer is correct; (e) allow students to use pictures or diagrams to support their thinking; (f) provide feedback about student think-aloud during guided and independent practice; and (f) if necessary, record and analyze student verbalizations to inform future instructions.

The research to date has tended to focus on verbalization and its relationship with students' impulsivity as well as self-regulation during problem-solving (National Mathematics Advisory Panel, 2008); (b) errors made while solving the problem (Parmar, 1992; Rosenzweig, Krawec, & Montague, 2011); (c) recalling relevant information (Nagro, & Cornelius, 2013); and (d) strategy acquisition (Schunk & Cox, 1986). There has been relatively little literature published on the effect of questioning, prompts, sentence starters, student knowledge/use of vocabulary on think-aloud strategies or verbalizations. Also, more attention should be paid to how the types and rigor of math tasks/word problems affect student verbalizations.

Table 3

Quality Indicators for Research Studies by Domain.

	Studies (2002-2011)					
Quality Indicators	Fuchs, Fuchs, Hamlett, & Appleton (2002)	Owen & Fuchs (2002)	Tournaki (2003)	Xin, Jitendra, & Deatline-Buchman (2005)	Rosenzweiz, Krawec, & Montague (2011)	Domain Average
Research Design	2	2	2	2	2	2
Quantity of research	2	2	2	2	2	2
Methodological quality	1	2	1	2	2	1.60
Magnitude of effect	2	2	2	2	2	2.00
Study Average	1.75	2	1.75	2	2	1.90

Table 3 continued

Quality Indicators	Studies (1986-2000) Manalo, Bunnell, & Stillman (2000)	Naglieri & Johnson (2000)	Hutchinson (1993)	Parmar (1992)	Schunk & Cox (1986)	Domain Average
Research Design	2	2	2	2	2	2
Quantity of research	2	2	2	2	2	2
Methodological quality	1	2	0	2	2	1.25
Magnitude of effect	0	2	2	2	2	1.50
Study Average	1.25	2	1.5	2	2	1.69

Note. The scores are based on a 3-point Likert-type. The scale ranges from 0 to 2: a score of 0 was assigned when the quality indicator was not met or not reported on; a score of 1 was assigned when the quality indicator was somewhat met; and a score of 2 was assigned when the quality indicator was completely met.

Table 4

Summary of Studies

Study	Population	Skill	Design & Duration	Outcome
Rosenzweig, C., Krawec, J., & Montague, M. (2011). Metacognitive Strategy Use of Eighth-Grade Students with and without Learning Disabilities during Mathematical Problem Solving: A Think-Aloud Analysis. Journal Of Learning Disabilities, 44(6-), 508-520.	Eight grade students with mathematical learning disabilities Three groups: math learning disabilities (14), low achieving in math (34), and high achieving in math (25)	Word problem	Randomized Control Trial (RCT) Cognitive-metacognitive intervention used: SOLVE IT! The researchers modeled the metacognitive/mnemonics strategy (SOLVE IT) using a think-aloud; the students verbalized their math thinking while they solved three word problems The researchers recorded, transcribed, and analyzed the verbalizations of 73	Student verbalization is an effective cognitive strategy Student verbalization (think-aloud) enhances short-term memory Student errors are revealed through think-aloud

			eighth-grade students while they solved math questions	
Tournaki, N. (2003). The differential effects of teaching addition through strategy instruction versus drill and practice to students with and without learning disabilities. Journal of Learning Disabilities, 36(5), 449-458	Third and fourth grade students with mathematical learning disabilities	Computation	Randomized Control Trial (RCT) 8 sessions of 15 minutes Strategy instruction with verbalizations The strategy instruction incorporated immediate corrective feedback, systematic review, and re-teaching whenever errors occurred.	Elementary students with MLD benefit more from strategy instruction with verbalizations than from instruction through drill and practice. Effect size: 1.61
Manalo, E., Bunnell, J., & Stillman, J. (2000). The use of process mnemonics in teaching students with mathematics learning disabilities. Learning Disability Quarterly, 23, 137–156	Eight grade students with mathematical learning disabilities	Word problems	Strategy instruction plus mnemonics vs. strategy instruction only Randomized Control Trial (RCT) 10 sessions of 25 minutes	Students in the strategy instruction plus mnemonics group performed higher than those in the strategy instruction only group. Also, students in the process mnemonics condition maintained a higher average performance six weeks following intervention.
Fuchs, L. S., Fuchs, D., Hamlett, C. L., & Appleton, A. C. (2002). Explicitly teaching for transfer: Effects on the mathematical Problem-Solving performance of students with mathematics disabilities. *Learning Disabilities Research & Practice,* 17(2), 90-106.	Fourth grade students with mathematical learning disabilities	Word problems	Randomized Control Trial (RCT) Problem-solving tutoring vs. basal instruction only 24 sessions of 33 minutes Researcher provided instruction/interventi on Interscorer agreement: 96-99%	Effect: size 1.78 Student verbalization that is preceded by an explicit instruction on a strategy or strategies that include teacher modeling think-alouds is an effective intervention for students with disabilities related to math
Owen, R. L., & Fuchs, L. S. (2002). Mathematical problem-solving strategy instruction for third-grade students with learning disabilities. *Remedial and*	3rd grade students with mathematical learning disabilities	Word problems	Randomized Control Trial (RCT) Explicit visual strategy instruction vs. control (basal	Effect size: 1.39 Students with MLD demonstrated improved achievement in math problem solving when

Special Education, 23(5), 268-278.			instruction) 6 sessions of 30 minutes Intervention/instruction provided by researcher Reliability Pre-Post Measures: 0.89 Students solved and explain the problem using visuals	they received strategy instruction with verbalization Student and teacher attitudes about the instructional strategy and were positive
Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word Problem—Solving instruction on middle school students with learning problems. *The Journal of Special Education, 39(3*), 181-192.	Middle school students with mathematical learning disabilities	Word problems	Randomized Control Trial (RCT) 12 sessions of 60 minutes Students were taught explicitly how to use visual representations/ schematic diagrams to display the solution for a word problem Explicit schema-based strategy instruction (SBI) Intervention/ instruction provided by researcher Reliability Pre-Post Measures: 0.88 Interscorer agreement: 100%	Random-Weight Effect: 2.15 Direct instruction enhanced the problem-solving skills and mathematics performance of students with learning disabilities in math (MLD). Schema-based instruction (SBI) group significantly outperformed the general strategy instruction (GSI) group on immediate and delayed posttests as well as the transfer test
Hutchinson, N. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. Learning Disability Quarterly, 16(1), 34-63.	Middle school ($8-10^{th}$ grade) students with mathematical learning disabilities	Word problem	Randomized Controlled Trial (RCT) Instruction provided by the researcher 60 sessions of 40 minutes Effect Size: 1.24	Student verbalization is an effective strategy for enhancing the performance of students with MLD Thinking aloud facilitates students' self-regulation during problem solving. Students with MLD can learned how to think aloud when teacher

				model the strategy Students with MLD that received explicit think-aloud instruction on the cognitive and metacognitive strategy of problem solving achieved more than the control group. Student and teacher verbalizations are similar, suggesting that students with MLD were able to recall and apply the strategy that teacher taught
Schunk, D. H., & Cox, P. D. (1986). Strategy training and attributional feedback with learning disabled students. Journal of Educational Psychology, 78(3), 201.	Middle school students with math learning disabilities	Computation	Randomized Control Trial (RCT) 6 sessions of 45 minutes Continuous student verbalizations vs. no student verbalizations The researchers instructed students to verbalize what they were thinking out loud while solving problems.	This instructional intervention resulted in an effect size of 0.07 There is a direct relationship between explicit teacher instruction, student verbalization, and student mastery of math concepts. Schunk & Cox (1986) established that verbalizing each step of the math process helped students with MLD remain focused and recall steps. Student verbalizations support strategy acquisition
Naglieri, J., & Johnson, D. (2000). Effectiveness of a cognitive strategy intervention in improving arithmetic computation based on the PASS theory. Journal of Learning Disabilities, 33(6), 591-597.	Middle school students with math learning disabilities	Word problem Computation	One experimental group and four contrast groups Students were taught the steps in the cognitive problem-solving strategy -- PASS (Planning, Attention, Simultaneous, Successive) Teachers used questions to probe	Naglieri and Johnson (2000) confirmed positive outcomes for students with MLD who verbalized a strategy while solving math problems. Students who experienced difficulties in planning benefited the most from the verbalization process Students with MLD

			thoughts if no students' verbalizations were heard after 5 seconds.	benefit from cognitive strategy instruction that emphasized planning. On the other hand, children without MLD (no planning weakness) who received the same intervention did not show the same level of improvement in math computation.
Parmar, R. (1992). Protocol analysis of strategies used by students with mild disabilities when solving arithmetic word problems. Diagnostique, 17(4), 227-243.	Students with MLD (ages 8-14)	Word problems	Randomized Control Trial (RCT) The researcher examined the types of errors 31 students with MLD made when solving math problems. Students verbalized their math reasoning/ thinking, and the teacher asked questions and took notes on student think-alouds and observable behaviors while solving the math problems The notes were transcribed and the errors were identified: skill deficit or knowledge deficit	Student verbalizations helped teachers to understand the thinking processes of students

CHAPTER 4: INTERVENTION PROCEDURES AND METHODOLOGY

This chapter discusses the research design, process evaluation design for examining the implementation of the intervention, and the outcome evaluation design for examining the proximal outcomes of the intervention.

Research Design

Randomized Controlled Trial (RCT) and Single-Case Experimental Designs (SCEDs) are commonly used to determine evidence-based practices and interventions in special education (Gersten et al., 2005; Horner et al., 2005). Several experts support the SCEDs because the design provides researchers with a "flexible and viable alternative to group designs with large sample sizes" (Smith, 2012, p. 1). In this regard, the dissertation research employs SCED due to few study participants (Smith, 2012). The students served as their own controls.

There are three commonly used single-case experimental designs: A-B-A-B Withdrawal (Reversal) Designs, Multiple-Baseline Designs, and Alternating Treatments Design (Smith, 2012). The reversal design and multiple baseline design are frequently used in special education research (Horner et al., 2005). The initial decision was to use the reversal design for this study. However, after a review of the research design by Dr. Kratochwill (a leading expert in the field of single-case design research) on May 31st, 2016, the research approach was changed to multiple baseline design. One of the reasons for this change is that reversal design involves the withdrawal of the independent variable (intervention), and the change in behavior caused by the intervention may not be reversed after the intervention is withdrawn. Hence, a causal relationship between the independent variable (IV) and dependent variables (DV) would be difficult to establish. A multiple baseline design involves staggering the introduction of the intervention across the participant. Usually, it includes at least six phases of baseline (A) and intervention (B) conditions (Byiers, Reichle, & Symons, 2012; Kratochwill et al., 2010; Kratochwill et al., 2013; Smith, 2012). Figure 6 illustrates the repeated and miniature AB pattern of multiple baseline design. The behavioral changes were evident for each participant following the introduction of intervention or treatment.

In this study, introduction of the independent variable (i.e., verbalization intervention/instruction) was staggered across four participants (fourth graders with MLD) in order to establish or demonstrate that changes in the dependent variables (i.e., conceptual understanding and procedural fluency) reliably happened only when the independent variable (i.e., verbalization intervention) was introduced. Because the baseline phase of each student did not begin simultaneously (i.e., begin at different points in time), the approach is called a non-concurrent multiple baseline design. If the multiple baselines are conducted across participants, Byiers and colleagues (2012) recommended that "all participants must be comparable in their behaviors and other characteristics." (p. 10). Table 6 summarizes the student characteristics.

Disagreement regarding the suitability of various approaches for analyzing SCED data has been ongoing for decades (Smith, 2012); however, visual inference of graphed data remains the standard by which SCED data are most commonly analyzed (Borckardt et al., 2008; Kratochwill et al., 2010, 2013; Parker, Cryer, & Byrns, 2006). Three parameters are important to visually presented data and graphs: trend, slope, and level. Some researchers have argued that statistical analysis in the context of SCED is unnecessary (Shadish, Rindskopf, & Hedges, 2008). In fact, Brossart, Parker, Olson, and Mahadevan (2006) found that only 20% of the published single-case studies they reviewed used statistical analysis for establishing a functional relationship between the IV and DV.

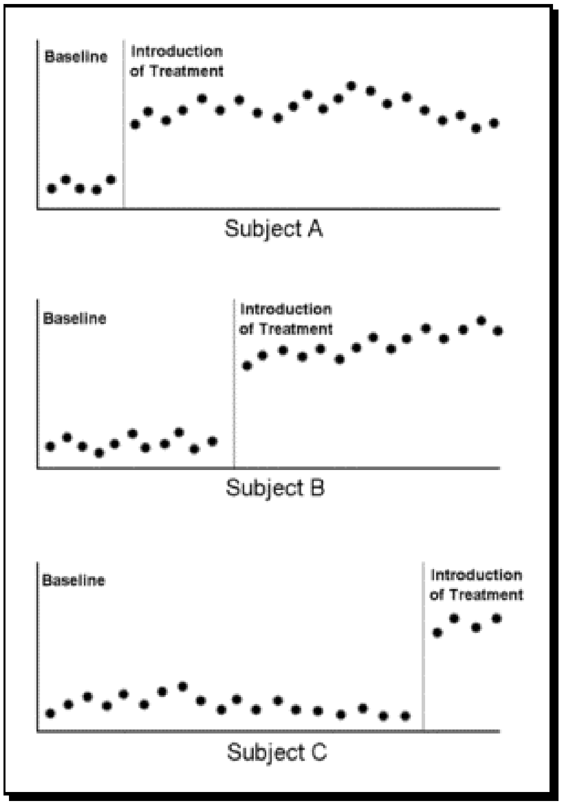


Figure 6. This figure illustrates the multiple baseline design. The behavioral changes were directly related to the treatment/intervention.

Hypothesized Outcomes

This study employs SCED method of analysis -- visual and statistical analysis -- to determine the effect of verbalizations on the mathematical proficiency of four participants (Parker, Cryer, & Byrns, 2006; Smith, 2013). Before the intervention was implemented, the researcher hypothesized that the visual and statistical analysis (e.g., slope, trend, level, effect sizes) of the SCED data would show an increased conceptual understanding and procedural fluency of all the study participants. Similarly, the intervention was projected to generate an effect size within the range (0.07 to 2.01) reported in the mathematics intervention literature.

Effect Size

There is no consensus on a standard effect size methodology within SCED (Ross & Begeny, 2014). Three approaches are often used to calculate effect sizes in SCED: standardized mean difference (SMD), percentage of non-overlapping data (PND), and Mean Baseline Reduction (MBR) (Olive & Smith, 2005). Other effect size methods include pairing regression and non-parametric approaches (Ross & Begeny, 2014). These approaches and their interpretations are summarized in Table 8. Researchers have reported consistent results across these different approaches (Campbell, 2004; Olive & Smith, 2005).

The present study employs the standardized mean difference method to calculate the effect sizes based on a recommendation from Kratochwill who reviewed the blueprint for the study (Hedges, Pustejovsky, & Shadish, 2013; Kratochwill et al., 2010; Kratochwill et al., 2013; Smith, 2012). Similarly, Hunt and Vasquez (2014), Maccini, Mulcahy, and Wilson (2007), Marita and Hord (2017), Methe, Kilgus, Neiman, and Riley-Tillman (2012), and Strickland and Maccini (2012) discuss numerous studies of mathematics interventions for students with MLD that employed the standardized mean difference method to calculate the effect sizes.

Process Evaluation (Fidelity of Implementation)

There are two elements of process evaluation that guided this dissertation study work: adherence and dose (Dusenbury, Brannigan, Falco, & Hansen, 2003; Saunders, 2005). "Dose is defined as the amount of program content received by participants" (Dusenbury et al., 2003, p. 241), while adherence (fidelity) is "the extent to which implementation of particular activities and methods is consistent with the way the program is written" (Dusenbury et al., 2003, p. 241). In this regard, the present study sought to answer the following process evaluation questions:

- **Process Evaluation RQ 1:** To what extent did the teachers adhere to the verbalization procedures during the intervention?

- **Process Evaluation RQ 2:** Did the students receive the intended level of treatment to promote mastery of the verbalization strategies?

Procedural fidelity is very important when implementing instructional interventions. Durlak and DuPre (2008) found that when interventions (implemented with high fidelity) yielded average effect sizes that are two to three times higher than interventions that were not implemented with fidelity. The present study operationally defines fidelity of implementation as the extent to which the teachers (implementation team) adhere to the verbalization intervention strategies (Dusenbury et al., 2003; O'Donnell, 2008; Nelson, Cordray, Hulleman, Darrow, & Sommer, 2012). In other words, fidelity of implementation involves: (a) the extent to which the verbalization intervention is implemented as designed; (b) how long (duration) it takes to implement the intervention before students master the strategies; and, (c) what the intervention looks like when it is implemented (quality). These components -- fidelity (quality), dose (delivered and received) -- are related to the critical elements for process evaluation plans described by Saunders (2005).

Conceptualizing fidelity of implementation involves "linking the intervention core components to outcomes." (Nelson et al., 2012, p. 375). Nelson and colleagues propose a model-based approach for conceptualizing fidelity of implementation and assessing implementation fidelity. Further, specifying the intervention model and identifying fidelity indices are two critical steps in conceptualizing the fidelity of implementation (Nelson et al., 2012). Based on this assertion, a simple theory of change (see Figure 7) that identifies the core intervention components and fidelity indices has been developed. Fixsen, Naoom, Blase, Friedman, and Wallace (2005) define core components as "the most essential and indispensable components of an intervention practice or program (core intervention components) or the most essential and indispensable components of an implementation practice or program (core implementation components) (p.24). Researchers need to conceptualize and measure the distinct intervention outcomes and implementation outcomes. In this regard, process evaluation would not only focus on the fidelity of the instructional intervention but also the fidelity of the activities (e.g., professional development sessions and observation cycles).

Describing what constitutes high-quality implementation is often a challenging task (Saunders, 2005). High-quality implementation depends, in part, on the fidelity of implementation. According to Sanetti and Kratochwill (2009), fidelity of implementation is the "extent to which essential intervention components are delivered in a comprehensive and consistent manner by an interventionist trained to deliver the intervention." (p. 448). The present study employs a fidelity checklist that aligns with the key components of the intervention. The fidelity checklist in Table 5 enumerates the intervention steps and procedures that the observer will look for during the observations.

Gresham and MacMillan (2000) and Schulte, Easton, and Parker (2009) noted that there is no conclusive guidance concerning the level of fidelity of implementation that is needed to achieve a substantial gain in student achievement. Nonetheless, Browder et al. (2008) and Horner et al. (2005) recommend that Single-Case Experimental Design studies must include adequate procedural fidelity above 80%. It is expected that the teachers who implement the intervention components with greater fidelity would have achieved better outcomes. For the present study, implementation fidelity of 80% or higher is acceptable, and teacher with implementation fidelity below 80% will receive additional training. Teachers will also receive support if they are uncomfortable with one or more of the intervention components.

Dusenbury et al. (2003) associate training and professional development with high-quality fidelity of implementation. Although the participants would acquire knowledge and skills during the professional development, fidelity of implementation can only occur when coaching (i.e., observations, feedback) is subsequently provided (Dusenbury et al. 2003; Fixsen et al., 2005; Joyce & Showers, 2002; Saunders, 2005). Research reveals that majority of teachers have difficulty maintaining high levels of fidelity for more than three weeks after starting an intervention or practice. Hence, coaching and ongoing support should be part of the intervention (Hagermoser-Sanetti, Fallon, & Collier-Meek, 2013).

Table 5

Fidelity checklist

#	Indicators	Yes	No	Note
	Introduction and Presentation			
1	Begin the lesson with a clear statement of the learning target(s).			
2	Review prior skills and knowledge before beginning instruction.			
3	Read the word problem twice			
4	Specify and model the steps in the THINK framework. • TALK about the problem. • HOW can it be solved? • IDENTIFY and use a strategy to solve the problem. • NOTICE how the strategy helped you solve the problem. • KEEP thinking about the problem. Does it make sense?			
5	• Verbalize thoughts (thinking aloud) as each step is completed. • Model how to self-question and think aloud while solving problems.			
6	Teacher uses: • manipulatives and visuals. • words, numbers, and/or pictures to make explanation clear.			
7	Teacher delivers instruction at an appropriate pace. The desired pace is neither so slow that students get bored nor so quick that they can't keep up.			

Guided Practice

8 Teacher reads the word problem twice.

9 Teacher and student apply the steps in the THINK framework and verbalizes reasoning related to the task.

10 Teacher and student use:
- manipulatives and visuals.
- words, numbers, and/or pictures to make explanation clear.

11 Teacher asks appropriate questions to probe student thoughts if no verbalizations after 5 second.

12 Teacher provides immediate affirmative and corrective feedback.

13 Teacher ask questions to check for student understanding

Independent Practice

14 Teacher reads the word problem twice.

15 Teacher asks appropriate questions to probe student thoughts if no verbalizations after 5 second.

16 Teacher reminds/encourages student to use:
- manipulatives and visuals.
- words, numbers, and/or pictures to make explanation clear.

Indicators of Fidelity of Implementation

Deriving indicators from the logic model (see Appendix D) involves differentiating the constructs underlying the intervention (Nelson, 2012). The logic model associated with the present study includes four main constructs: (a) teacher ability to describe the verbalization strategies; (b) teacher ability to effectively model and implement verbalization strategies; (c) student ability to verbalize their reasoning while solving word problems; and (d) student ability to understand and solve the word problems correctly. A brief explanation is provided below.

Teacher ability to describe the verbalization strategies. The teachers participating in this study will be able to describe or explain the specific steps required to facilitate student verbalizations in the classroom. These steps are: (a) explicitly teach the steps in the strategy; (b) model the steps by verbalizing thoughts (thinking aloud) as each step is completed; (c) use appropriate questions, probes and prompts to enhance student verbalizations; (d) allow students to practice thinking aloud as they independently complete the word problems; and (e) provide corrective feedback to students and fade support as they master the strategy (Hutchinson, 1993; Naglieri & Johnson, 2000; Rosenzweig, Krawec, & Montague, 2011; Schunk and Cox, 1986; Tournaki, 2003).

Teacher ability to effectively model and implement verbalization strategies. Teacher verbalization helps students with MLD gain an understanding of how a math problem can be approached and solved (Montague, 2008). In research studies, student verbalization (metacognitive strategy) is often prefaced by a direct or explicit instruction on a cognitive strategy (e.g., mnemonics) that include teacher think aloud (Naglieri & Johnson, 2000; Rosenzweig et al., 2011; Schunk and Cox, 1986; Tournaki, 2003). The teachers participating in this study will be able to effectively model and implement the verbalization strategies while solving problem to students. The indices associated with this indicator are: (a) self-report surveys, (b) face-to-face interviews, (c) direct observations with fidelity checklist, and (d) video-recording for teachers to review and reflect on their teaching practices.

Student ability to verbalize their reasoning while solving word problems. Research studies that established that students with MLD could verbalize their reasoning while solving word problems if the teachers effectively model the strategies (Hutchinson, 1993; Naglieri & Johnson, 2000; Rosenzweig et al., 2011; Tournaki, 2003). Students with MLD will be instructed on how to use the strategy before they are asked to verbalize their thinking as they solve problems using the strategy. A study conducted by Schunk and Cox (1986) established a direct relationship between explicit teacher instruction, student verbalization, and improved student achievement. The indices associated with this indicator are (a) student interview, (b) classroom observations, (c) student work samples, and (d) video-recording of student verbalizing their reasoning.

Student ability to understand and solve the word problems correctly (conceptual understanding and procedural fluency). Research has established that the mathematical errors that students with MLD demonstrate are related to the difficulties they experience while problem-solving (decision-making), retrieving numerical facts, capturing the meaning of the basic arithmetic operation symbols, etc. (Geary et al., 2007; Gersten et al., 2008). However, these students will be able to understand and solve word problems correctly (minimize errors) if they are given the opportunity to verbalize their thinking processes (Cole & Wasburn-Moses, 2010). Process evaluation data were collected using surveys, fidelity checklists, direct observations, face-to-face interviews, and student work samples (Nelson et al., 2012).

Student work samples were used to determine students' mathematical proficiency (i.e., conceptual understanding and procedural fluency). The conceptual understanding and procedural fluency were evaluated using a rubric that measured their ability to understand, to plan, to solve, and to check each problem (see Table 14). The students received scores that reflect their conceptual understanding -- ability to understand the problem and show workable plan to solve the problem -- and procedural fluency -- correct implementation of the plan and ability to give a reasonable solution (Rittle-Johnson & Schneider, 2014; Thomas, 2006). Fidelity of implementation data was collected per teacher. The minimum level of fidelity required is 80%.

Outcome Evaluation

This dissertation study addresses an overarching outcome evaluation question:

RQ 1: To what extent does the verbalization intervention affect the mathematical problem solving of fourth graders with mathematical learning disabilities (MLD)? The mathematical problem-solving ability of students was investigated based on two areas: (a) conceptual understanding, that is, student ability to comprehend mathematical concepts, operations, and relations and (b) procedural fluency, that is, student ability to carry out procedures flexibly, accurately, efficiently, and appropriately (Geary, 2004, 2011; Jitendra, DiPipi, & Perron-Jones, 2002; Karagiannakis et al., 2014; Watson & Gable, 2012). The impact of student verbalizations on these two major strands of mathematical proficiency are elaborated in chapter 5.

Participant Characteristics and Setting

This study was conducted at a public charter school in an east coast metropolitan area. The school serves approximately 1000 students in prekindergarten through 12th grade. These students represent a broad range of ethnic and socioeconomic backgrounds: Hispanic - 47%, Black - 37%, White - 9%, others - 7%, low income - 73%, special education - 14%, and limited English proficiency - 19%. At this time, approximately 12% of the student population meet the IDEA eligibility criteria for Specific Learning Disabilities (SLD) in mathematics, reading, and writing. The school addresses the needs of its special education population through an inclusion program. Academic and related services are provided to students with disabilities within the regular classroom by a team consisting of inclusion teachers responsible for each classroom, a school psychologist, a counselor, an occupational therapist, and a speech and language pathologist.

Four fourth graders (three females and one male) with MLD with an average age of nine years participated in this study because they met the inclusion criteria. All these students have been found eligible as children with Specific Learning Disabilities (SLD) in mathematics. In order words, they met eligibility criteria for MLD because prior to their enrollment in the special education program, they did not make sufficient progress to meet age or state-approved grade-level standards in mathematics when provided with learning experiences and instruction appropriate for the student's age or state-approved grade-level standards. Further, the students demonstrated a discrepancy between achievement (as measured by the academic evaluation) and measured ability (as measured by the intellectual evaluation) of two years below chronological age and/or at least two standard deviations below the student's cognitive ability as measured by appropriate standardized diagnostic instruments and procedures. The verbalization intervention focuses on fourth graders with MLD because significant math deficits are "clearly established, and identification of mathematics disabilities typically begins in 3rd grade" (Fuchs et al., 2008, p. 2).

Two of the students are English language learners (ELLs). As described in the Elementary and Secondary Education Act of 1965, this definition classifies an English learner any student: (a) who is aged 3 through 21; (b) who is enrolled or preparing to enroll in an elementary school or secondary school; (c) who was not born in the United States or whose native language is a language other than English; and who comes from an environment where a language other than English is dominant; and (d) whose difficulties speaking, reading, writing, or understanding the English language may be sufficient to deny the individual the ability to achieve successfully in classrooms where the language of instruction is English.

Assessing Comprehension and Communication in English State to State for English Language Learners (ACCESS for ELLs) is a large-scale English language proficiency test for K-12 Students. The test was developed by Wisconsin Center for Education Research (University of Wisconsin-Madison) in partnership with the Center for Applied Linguistics. This yearly assessment is used to monitor student progress in English language proficiency. It is also used as a criterion to determine when ELL students have attained full English language proficiency. The English language proficiency scores (2017 ACCESS for ELLs) are included in Table 6. The ELLs participating in this study are categorized as "developing." This means that the students are able to process, understand, produce, or use: (a) general and some specific language of the content areas; (b) expanded sentences in oral interaction or written paragraphs; and; (c) oral or written language with phonological, syntactic, or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative, or expository descriptions with sensory, graphic, or interactive support (Wisconsin Center for Education Research). Students are considered fully proficient when they are able to process, understand, produce, or use oral or written communication in English comparable to proficient English peers. According to 2015 and 2016 PARCC tests in mathematics, none of these students achieved proficiency in mathematics. The characteristics of the participating students are presented in Table 6.

Table 6

Student Characteristics

Student	Gender	Age	Race	ELPL	Standardized assessment results
B1	M	9 Years 9 Months	AA		Below average in mathematics
B2	F	9 Years 2 Months	H	Developing (3.8)	Below average in mathematics
A2	F	9 Years 6 Months	H	Developing (3.9)	Below average in mathematics
A1	F	9 Years 4 Months	AA		Below average in mathematics

Note. AA = Black or African American, H= Hispanic/Latino, ELPL = English language proficiency level

The intervention team includes two special education teachers, a mathematics intervention specialist, and the student investigator (researcher). All the team members have master's degrees in education. The two teachers have been working with children with disabilities for an average of five years. The characteristics of the participating staff are presented in Table 6.

Table 7

Staff Characteristics

Team Member	Gender	Highest Degree	Race	Expertise	Years of teaching experience
JH	F	Master's	W	Math Intervention	9
ET	M	Master's	AA	Special Education	10
DW	F	Master's	W	Special Education	3
YQ	F	Master's	H	Special Education	10

Note. AA = Black or African American, W= White, H= Hispanic/Latino. The participant recruitment documents are included on the appendix section of this report.

Procedure (Based on Logic Model)

Figure 4.2 depicts the simple theory of change and Appendix D shows the logic model associated with the dissertation research. The key components of the intervention may be related to the positive learning outcomes of the participants.

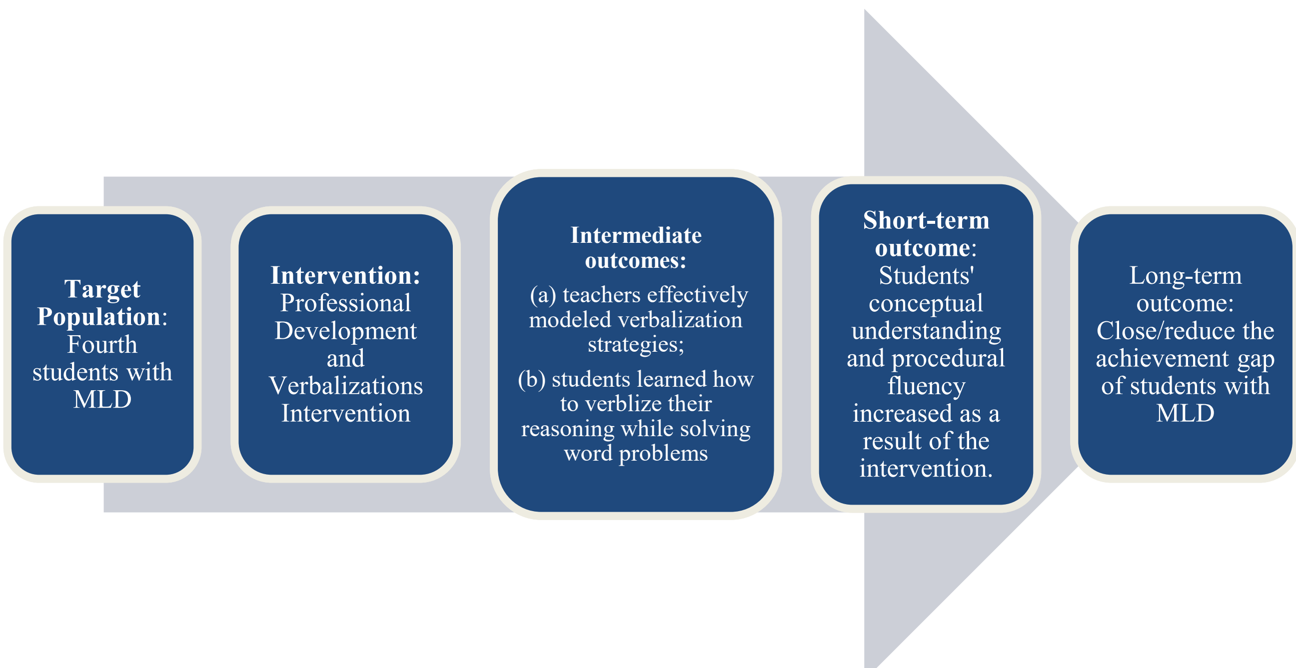


Figure 7. A diagram of simple theory of change depicting the components and activities that influence the intermediate and short-term outcomes.

Professional development/training. Research has linked high-quality professional development to effective classroom instruction and improved student learning (Borko, 2004; Garet, Porter, Darling-Hammond & Bransford, 2005; Desimone, Birman, &Yoon, 2001; Rosenzweig, Krawec, & Montague, 2011). According to Garet and colleagues (2001) and Desimone (2009), high-quality professional development includes the following core features: (a) focus on content to be learned by students; (b) active (hands-on) learning during the professional development experiences; (c) coherence of the learning to teachers' professional needs; (d) occurrence over a long duration; (e) collective participation by educators in their professional learning; and (f) individual follow-up through supportive observation and feedback. These core features guided the development and implementation of the four professional development sessions for the teachers. Chapter 5 describes additional information about the contents and implementation of the professional development sessions.

Inputs, resources and infrastructure. The inputs required to plan and execute the professional development are (a) needs assessment results; (b) adequate materials, equipment, and facilities to ensure full participation of teachers; (c) teacher participation; (d) time in school schedule; (e) existing knowledge of teachers; (f) approval by the principal (executive sponsor); (g) knowledge of single case experimental design research; and (h) knowledge of verbalization intervention research. The needs assessment study revealed that lack of evidence-based instructional practices is one of the underlying causes and factors associated with the underachievement in mathematics of students with MLD within the organization. This significant insight informed the intervention -- helping teachers to develop the skills required to model and facilitate student verbalizations (evidence-based practice) of their mathematical reasoning while solving word problems. Students with MLD demonstrated improved achievement in problem-solving when they received strategy instruction with teaching modeling verbalizations (Fuchs et al., 2002; Rosenzweig, Krawec, & Montague, 2011; Tournaki, 2003).

The inclusion office conference room was used for the professional development sessions. The room is spacious and well ventilated. It is also equipped with a projector, table, chairs, and white board. Before the professional development sessions, the teachers were encouraged to read a few research articles (see Table 4) that describe how to model verbalization strategies.

Outputs. According to the logic model, the output of an intervention includes the activities and participation. The direct products of the professional development sessions include: (a) learning activities; (b) six hours of professional development sessions; (c) eight hours of observation cycles (observation, debrief/feedback) using the fidelity checklist; (d) two Special Education Teachers; (e) Mathematics Intervention Specialist; (f) Student Investigator (Researcher); (g) four students with MLD; and (h) research-based contents of the professional development. Similarly, the direct products of the verbalization intervention are student work samples and audio files of student verbalizations. An in-depth explanation of the outputs is explained in chapter 5.

Activities. As mentioned earlier, teacher knowledge of the verbalization strategies can greatly influence the academic achievement of students with MLD. In this regard, it is imperative to develop and implement high-quality professional development sessions (Desimone, 2009; Garet et al., 2001). In order for the professional development to be high-quality, the contents must include specific components recommended by mathematics intervention experts: (a) specify the steps in the strategy (Rosenzweig, Krawec, & Montague, 2011); (b) verbalize reasoning (think aloud) as each step is completed (Hutchinson, 1993; Naglieri & Johnson, 2000; Rosenzweig, Krawec, & Montague, 2011; Schunk and Cox, 1986); (c) use probes and prompts to enhance student verbalizations (Naglieri and Johnson, 2000); (d) allow students to practice verbalizing their thinking as they independently complete the tasks or word problems (Hutchinson, 1993; Naglieri and Johnson, 2000; Rosenzweig, Krawec, & Montague, 2011; Schunk & Cox, 1986); and (e) provide corrective feedback to students and fade support as the students master the strategy (Hutchinson, 1993; Schunk and Cox, 1986; Tournaki, 2003).

One of the core activities is observation cycle. Each observation cycle includes: (a) pre-meeting and goal setting, (b) mini-observations/lower-stakes visits, (c) debrief after the lesson or observation, and (d) ongoing support. The debrief agenda focuses on (a) reflecting on evidence of student learning; (b) making connections and brainstorming answers to anticipated and unanticipated challenges faced while implementing the verbalization intervention; (c) reviewing the observation/fidelity checklist; and (d) planning next steps. Also, student verbalizations were audiotaped and supported teacher reflection after the lessons.

Hagermoser Sanetti, Fallon, and Collier-Meek (2013) noted that teachers who received corrective feedback demonstrated higher implementation fidelity compared to those teachers who did not. This practice resulted in more positive outcomes for their students. Just as giving teachers feedback about their implementation of the verbalization intervention is essential for their professional development, receiving feedback from teachers about the is also important. Thus, teachers were given the opportunity to share their feedback about the implementation of the verbalization intervention. This feedback will be used to fine-tune the intervention and subsequent trainings.

Outcomes. According to the logic model, outcomes are the specific changes in program or intervention participants' behavior, knowledge, or skills. The model includes short-term, medium-term, and long-term outcomes. It is recommended that the short-term outcomes should be attainable within 1 to 3 years, while longer-term outcomes should be achievable within 4 to 6 years. In relation to intermediate outcomes, the professional development and observation cycles are expected to result in an acquisition of skills and knowledge required to facilitate student verbalizations while solving different word problems. An acquisition of this skill would contribute to teacher quality. Rivkin, Hanushek, and Kain (2005) found that teacher quality alone could account for 7% of the variance in student achievement gains. The short-term outcome aimed at helping students develop conceptual understanding and procedural fluency. The long-term outcome is to reduce or close the achievement gap of children with MLD.

Table 8

Effect Size Calculation in Single-Case Experimental Designs

	Standardized Mean Difference (SMD)	Percentage of Non-Overlapping Data (PND)	Mean Baseline Reduction (MBR)
Formula	Subtract the mean of the baseline phase from the mean of the intervention phase	Identify the highest baseline point	Subtract the average of last 3 intervention points from the average of the last 3 baseline points
	Then divide by standard deviation pooled across baseline and intervention phases Cohen's d = Mean (intervention) – Mean (baseline) / Pooled SD (intervention + baseline)	Count the number of intervention points that exceed the highest baseline point (non-overlapping).	Divide by the average of the last 3 baseline points
		Calculate the proportion of non-overlapping to total number of intervention points (can't use if baseline has a zero point)	Multiply by 100 for percent of baseline reduction (MBR)
Interpretation	d= 0.2 is a small effect, d=0.5 is a medium effect, and d=0.8 is a large effect	90%+ = Highly Effective	Relative effect: 200% is greater than 100%
		70%-90% = Moderately Effective	There is no standard for magnitude of effect.

	50%-70% = Minimally Effective
	>50% = Ineffective

Note. The Standardized Mean Difference (SMD) for single-case experimental design is not the same metric as the SMD for RCTs.

Strengths and Limitations of Design

Findings of causality depend on the internal validity of the research design. There are extraneous variables and factors that could jeopardize internal validity in SCED: history, maturation, instrumentation, statistical regression, testing, and attrition. These factors affect the demonstration of the functional relationship between the independent and dependent variables. Although RCTs are often seen as the "gold standard" because their procedures minimize the threats to internal validity threats, SCED is a viable alternative (Horner et al., 2005; Kratochwill & Levin, 2010; Shadish et al., 2002). In a group design, random assignment of participants controls for most of the threats to internal validity. In a SCED, however, repeated baseline measurements address most of the threats to the internal validity of the design. The repeated measurement helps because patterns illustrative of the threats often appear in the baseline (Parker et al., 2006; Smith, 2012). Another major limitation of SCED is external validity. It refers to the confidence others may have that the same intervention would yield similar results in similar studies, settings, or participants. "External validity of results from single-subject research is enhanced through replication of the effects across different participants, different conditions, and/or different measures of the dependent variable." (Horner et al., 2005, p. 171).

There are two major benefits of the multiple-baseline designs: (a) the approach does support the reversal of desired behaviors (intervention outcomes); and (b) because the replication of the experimental effect is across participants, the approach does not require the withdrawal of the independent variable (intervention). This is one of the major reasons the design appeals to most (approximately 70% of SCED) researchers and experts (Smith, 2012). Conversely, there are limitations associated with the multiple-baseline design approach: (a) because the replication of the experimental effect is across participants, other participants must wait before receiving the intervention; (b) the design cannot be used when the intervention can be applied to only one individual, behavior, and setting; and (c) generalization effects and excessive variability (during the baseline phase) may cause threats to internal validity. In order to ensure stability (i.e., limited variability) during the baseline phase, the researcher collected sufficient data. It has been recommended that the data from the baseline phase should be collected during three to twelve observations or intervention sessions (Parker et al., 2006; Kratochwill et al., 2010; Smith, 2012).

CHAPTER 5: IMPLEMENTATION, FINDINGS AND DISCUSSION

Introduction

This chapter describes the (a) the procedures followed during the professional development and the verbalization interventions; (b) the fidelity of implementation methods and results for both professional development and the intervention sessions; and (c) the methods and results of the intervention. Table 9 displays the timeline of activities. The core activities include professional development/training, observation cycles, and mathematics intervention. These principal activities components are depicted in Figure 8.

Table 9

Timeline of Activities

Time	Activity
March 3	IRB application approved
March 9	Recruitment and parental consent received
March 10	Student investigator met with the implementation team to discuss the intervention plan
March 13 - 17	Four professional development sessions
March 20	Baseline data collection started for all students
March 29	Verbalization intervention started for two students
March 28 – June 10	Observation cycle and ongoing support for teachers
April 12	Verbalization intervention started for the remaining two students
June 15	Intervention concluded
June 15 – August 15	Data Analysis

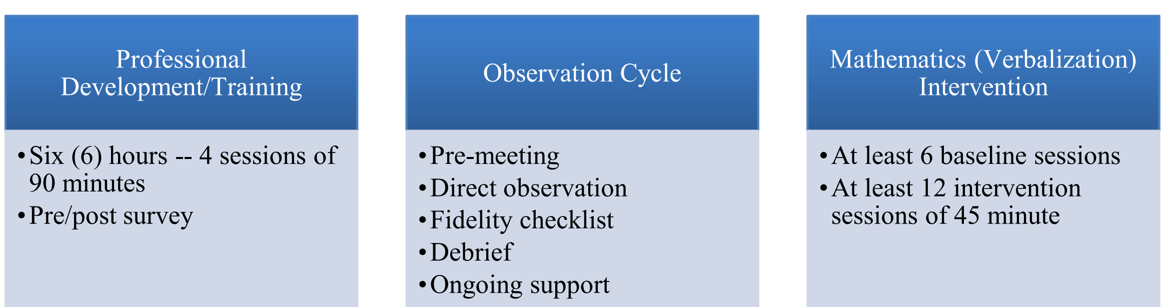


Figure 8. This figure illustrates the principal components of the intervention.

Professional Development

Table 10 presents an overview of the professional development/training activities. The teachers attended four sessions of ninety minutes onsite professional development/training. The room was well ventilated and equipped with a projector, table, chairs, and white board. Before the professional development sessions, the teachers were encouraged to review the meta-analyses of mathematics interventions for students with learning disabilities conducted by Gersten et al. (2009) and Baker et al. (2002). This activity was intended to acquaint the teachers with the research principles and justification for the verbalization intervention.

Several researchers have examined the effects of different characteristics of professional development on teachers' learning and change in classroom practices (Desimone, 2009; Garet et al., 2001; Joyce & Showers, 2002). The design and implementation of the professional development for teachers that participated in this study were informed by the work of Garet et al. (2001) and Desimone (2009). These researchers urged professional development designers to consider the following principles and activities while planning and implementing professional development for teachers: (a) focus on the content to be learned by students; (b) include active (hands-on) learning (c) align the training to teachers' professional needs; (d) engage the educators in their professional learning; and (e) provide an ongoing support through observation and feedback.

Table 10

Professional Development/Training Overview

Session 1	Session 2	Session 3	Session 4
Pre-training survey Introduction	Review session 1 Fostering algebraic thinking through questioning	Review session 2 Model verbalization using the THINK Framework	Review session 3 Model verbalization using the THINK Framework
Discuss the principles of Tier II/III Intervention	Math videos (teacher and student verbalizations)	Watch and debrief teacher verbalizations using the THINK framework	Teacher practice and feedback
Unpack Operations and Algebraic Thinking standards (i.e., 2.OA & 3.OA)	Introduce the components of the verbalization intervention	Break	Break

Break	Break	Teacher practice and feedback	Discuss the fidelity/implementation checklist
Sort/unpack problem types (i.e., result unknown, change unknown, start unknown)	Discuss/watch videos on how to use questions/probes and prompts to enhance student verbalizations	Questions and answer time	Discuss the student work rubric
What does mastery mean?	Model/role play with critique and feedback		Reiterate the key components of the intervention
Questions and answer time	Teacher practice		Questions and answer time
	Questions and answer time		Post-training survey

A pre-training survey administered on the first day of the professional development required the teachers to rate their knowledge and skills in the following areas: (a) knowledge of algebraic thinking standards; (b) knowledge of questioning techniques; (c) knowledge of the key principles of tier II/III; (d) knowledge of how to use questions to elicit student thinking and verbalizations; and (e) how to model best practices and implement strategies for children who struggle in mathematics. The post-training survey included the same questions as the pre-training survey. Both surveys used a five-point Likert scale (1=strongly disagree - 5 = strong agree).

The Response to Intervention (RTI) was introduced during the first session of the professional development because the verbalization intervention/procedures align with the principles of Intensive Interventions, a key component of the RTI framework. RTI includes the “concept of increasingly intense interventions to ensure that students receive sufficient learning opportunities to optimize their successful learning and achievement." (Mellard, McKnight, & Jordan, 2010, p. 217). Similar to RTI Tier III, the students that participated in this study received individualized, intensive interventions that target the mathematics skill deficits.

Teacher Knowledge of the Problem Types

Session one of the professional development focused on content to be learned by the students. Teachers participated in a sorting activity (led by the Mathematics Specialist) to unpack the two operations and algebraic thinking standards (i.e., 2.OA & 3.OA) and problem types. This activity required the teachers to place or assign certain word problems in the chart (see Tables 5.3 and 5.4). As part of the PD training, participants discussed their reflections of the process and considered the implications for student learning. To solidify their conceptual understanding of different problem types, the teachers also reviewed the charts below. All the word problems align with the Common Core State Standards.

Table 11

Algebraic thinking standards (i.e., 2.OA) problem types

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there we five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. There were three apples. How many apples were on the table before? ? – 2 = 3
Put together/take apart	Total Unknown Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Addend Unknown Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Both Addends Unknown Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
Compare	Difference Unknown ("How many more?" version): Lucy has two apples. Julie has five apples. How	Bigger Unknown (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How	Smaller Unknown (Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How

many more does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Note. The problem types displayed in Table 11 align with the Common Core Standards 2.OA.A.1. The standard requires students to use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Source: Common Core State Standards Initiative (NGA, 2010).

Table 12

Algebraic thinking standards (i.e., 3.OA) problem types

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	3 x 6 = ?	3 x ? = 18, and 18 ¸ 3 = ?	? x 6 = 18, and 18 ¸ 6 = ?
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example:* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example:* you have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example:* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example:* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example:* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example:* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example:* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is three times as much as a blue hat costs. How much does a blue hat cost? *Measurement example:* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example:* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?

General	*a x b = ?*	*a x ? = p, and p ¸ a = ?*	*? x b = p, and p ¸ b = ?*

Note. The problem types displayed in Table 12 align with the Common Core Standards 3.OA.A.3. The standard requires students to use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Source: Source: Common Core State Standards Initiative (NGA, 2010).

Session Two focused on one of the key elements of the verbalization intervention: using questions and prompts to elicit student thinking and verbalizations. The teachers were introduced to the five categories of question that can be used to foster algebraic thinking: managing, clarifying, orienting, prompting mathematical reflection, and eliciting algebraic thinking (Driscoll, 1999). For instance, orienting questions (e.g., what's the problem asking you to find?) are "intended to get students started, or to keep them thinking about the particular problem they are solving; may suggest ways to focus on the problem." (Driscoll, 1999, p. 6).

Participants watched and discussed two video clips that illustrate different verbalization strategies and questioning techniques that teachers can use to help students clarify and share their mathematical thinking in the classroom. The first video focused on the "revoicing" strategy, a simple strategy used by teachers used to clarify student verbalizations. The second video clip focused on the "say more" strategy. A teacher using this talk moves type might say, "Can you give me an example of ….?" (Anderson, Chapin, & O'Connor, 2011). At the end of the two videos, the training participants were debriefed about their experience. This part of the training was facilitated by the math specialist and student researcher using questions such as: What did you see happening in the videos? Did anything surprise you, interest you, or make an impression on you? What benefits do you see in these clips for the teacher and students?

The THINK framework (Thomas, 2006; Van de Walle, Karp, & Bay-Williams, 2013) was adopted as the template for teacher/student verbalizations and was introduced during the third session. Figure 9 depicts the THINK framework. To support the teachers to learn and understand it, the researcher modeled and verbalized the steps in the THINK framework while solving word problems for the teachers. Additionally, the teachers watched a 5-minute video on how to verbalize their reasoning while using the THINK framework. Later, the teachers practiced the steps and received feedback about their behaviors.

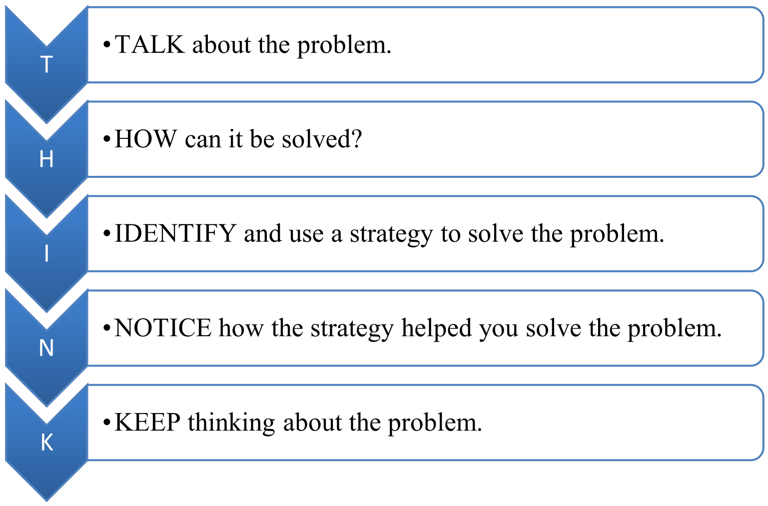


Figure 9. THINK: A framework for improving problem solving

The fourth session reinforced and built on the activities of session three. The teachers had the opportunity to practice and verbalize the steps in the THINK framework while solving word problems. The word problems (operations and algebraic thinking) were similar to the ones students were asked to solve during the instructional intervention sessions. Joyce and Showers (2002) assert that teachers' knowledge and skills often increase when the professional development includes discussion of the theory, modeling, and opportunities to practice the new instructional techniques or strategies. Further, feedback is an essential component of a high-quality professional development (Leach & Conto, 1999). In this regard, the mathematics specialist and student researcher provided objective feedback that targeted specific behaviors related to the verbalization strategies and word problem types. Research suggests that the targeted and timely feedback could help teachers develop the skills required for an effective instruction (Leach & Conto, 1999; Noell et al., 2005). Specifically, Noell and colleagues (2005) found that the students of teachers who received feedback demonstrated greater outcomes than students whose teachers did not receive feedback. It is expected that the feedback would contribute to the fidelity of implementation of the evidence-based instructional practices.

The professional development participants reviewed and used the fidelity checklist (see chapter 4) and student work rubric (see Table 14). The rubric, based on Pólya’s problem-solving process, was developed by Thomas (2006), the originator of the THINK framework. This problem-solving scoring rubric highlights four problem-solving skills and evaluates the students’ ability to understand, plan, solve, and check their solution to the problem. The students would earn a holistic score that reflects their conceptual understanding (understand and plan) and procedural fluency (solve and check). The student researcher answered all questions about the rubric posed by the teachers.

At the conclusion of the final PD session, the teachers completed the post-training survey. Figures 10, 11, and 12 compare the results of the pre- and post-training surveys. The teachers reported feeling more confident about core aspects of the instructional intervention/procedures after the professional development, especially in the following areas: (a) knowledge of algebraic thinking standards; (b) knowledge of questioning techniques; (c) knowledge of the fundamental principles of RTI (Tier II/III); (d) knowledge of how to use questions to elicit student thinking and verbalizations; and (e) modeling best practices/strategies for students.

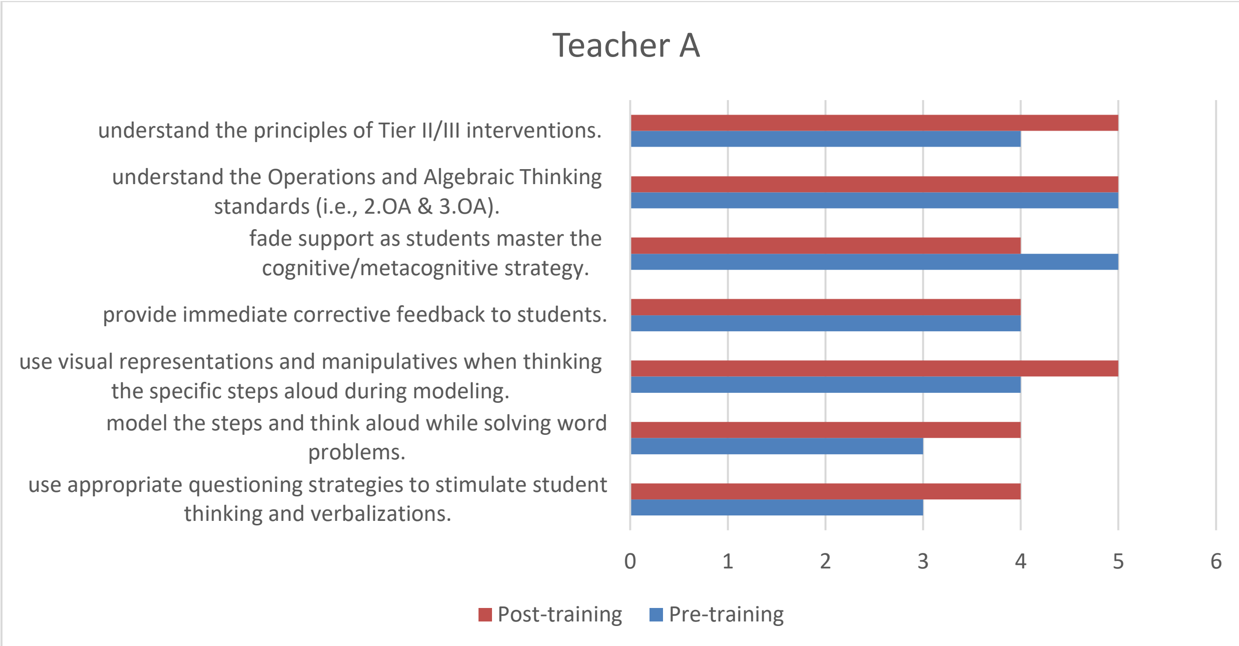


Figure 10. This figure illustrates the results of self-reports about teacher A perceived knowledge of key intervention components/procedures.

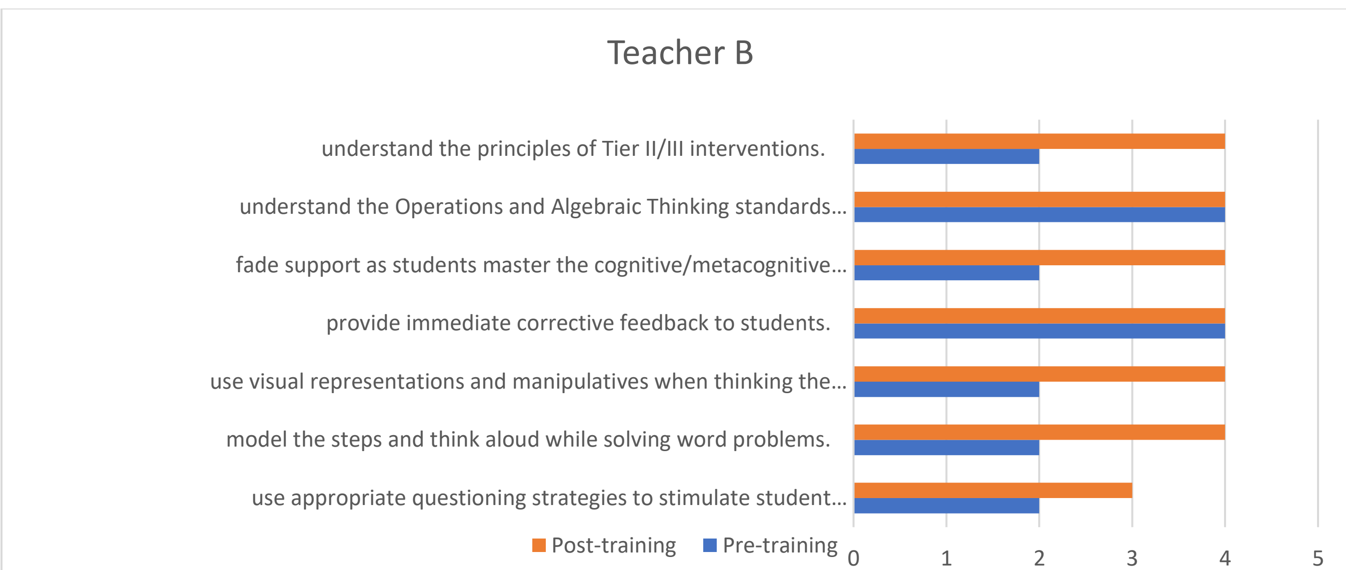


Figure 11. This figure illustrates the results of self-reports about teacher B perceived knowledge of key intervention components/procedures.

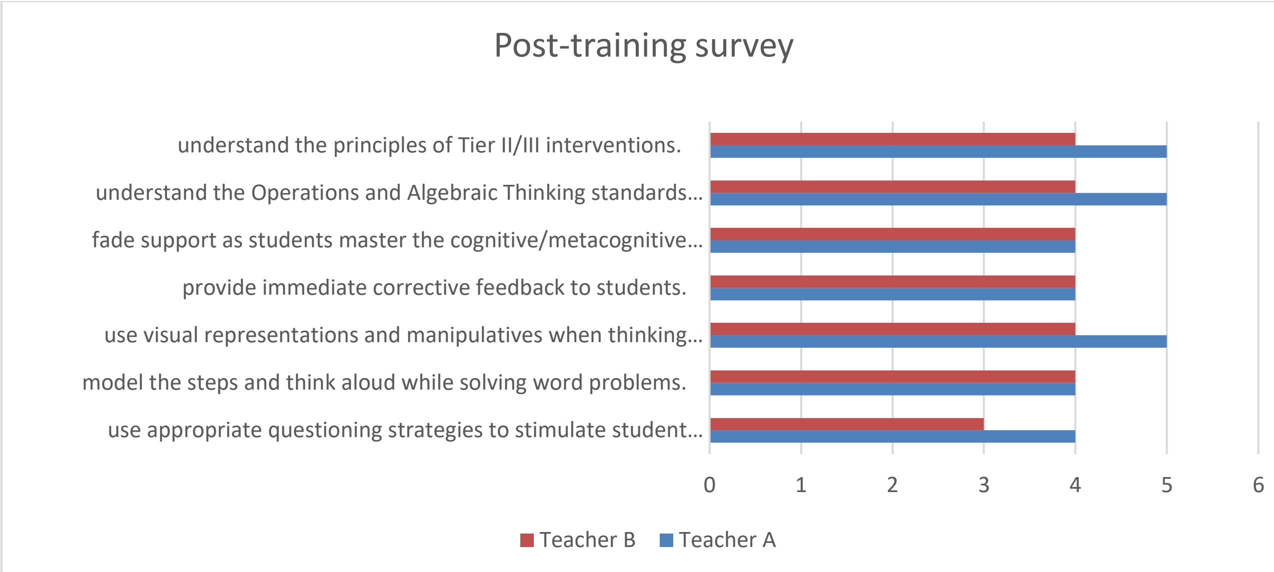


Figure 12. This figure illustrates the results of self-reports about teacher A and B perceived knowledge of key intervention components/procedures after the training.

Student Recruitment

The initial recruitment contact of parents/caregivers was done by one of the teachers participating in the study. The teacher teaches 2nd-graders and does not directly teach any of the students participating in the study. The parents or caregivers of eligible children were invited for a face-to-face conversation with the researcher if they signified interest in the study. During the face-to-face recruitment discussion, the parents had the opportunity to review the consent form and ask questions. After the parental consent had been obtained, the researcher met with the students to discuss the study using simple language. The student participants are not proficient readers but are still capable of assenting. They are reading (comprehending texts) on 1st-2nd-grade level, and therefore, an oral assent was preferred.

The Principal Investigator reviewed the oral assent script with the student researcher. Also, one of the teachers who observed the discussion confirmed that assent was not coerced and that it was given (see Appendix E for the oral assent script). The following procedures were implemented to avoid participant coercion or undue influence: (a) participants and the participants' parents/caregivers were informed that participation was completely voluntary; (b) the participants and their parents/caregivers were advised of their right to withdraw from the study at any time; (c) students were assured that their decision would not affect their grades. Additionally, the researcher informed the students that there was no guarantee that the intervention would help them with their mathematics learning.

Data Collection and Procedures

The intervention started after receiving approval from the Institutional Review Board, research setting, and all the required permissions from the parents/caregivers and students were obtained. The study was implemented over the course of three months (See Table 9). The baseline data collection phase started after the two teachers participated in four professional development sessions of ninety minutes each. Teacher A was randomly assigned to students A1 and A2, while Teacher B was assigned to students B1 and B2. All the students began participating in the study at the same time while at least six baseline data points were collected for each. After the baseline phase, the students received up to twelve intervention sessions of forty-five minutes each.

Consistent with a multiple baseline approach and staggered start points, two participants, students A1 and B1 started receiving the intervention while the two others, students A2 and B2, continued to receive only traditional instruction that did not include verbalization training. The verbalization instructions/interventions for students A2 and B2 started after four intervention data points were collected for students A1 and B1. Table 13 depicts the major similarities and differences between the baseline and intervention phases. During the traditional instruction, baseline data were collected.

Table 13

Description of guidance provided to teachers about the steps to take during the traditional/ baseline data collection sessions and the intervention sessions.

	Traditional (Baseline Data Collections Sessions)	**Intervention Sessions**
Introduction (Teacher)	• Start the lesson with a clear statement of the learning target(s). • Review prior skills and knowledge before beginning instruction. • Read the word problem twice	• Start the lesson with a clear statement of the learning target(s). • Review prior skills and knowledge before beginning instruction. • Read the word problem twice
Presentation (Teacher)	• Solve the problem • Use manipulatives and visuals • Use words, numbers, and pictures to make your explanation clear. • Deliver instruction at an appropriate pace. • The desired pace is neither so slow that students get bored nor so quick that they can't keep up.	Model (apply and verbalize) the THINK framework: • TALK about the problem. • HOW can it be solved? • IDENTIFY a strategy to solve the problem. • NOTICE how the strategy helped you solve the problem. • KEEP thinking about the problem. Does it make sense? • Use manipulatives and visuals • Use words, numbers, and pictures to make your explanation clear. • Teach students how to self- question and think aloud while solving the word problems. • Teachers use the "Developing Mathematical Thinking with Effective Questions aligned to the Mathematical Practices" document. • Deliver instruction at an appropriate pace. • The desired pace is neither so slow that students get bored nor so quick that they can't keep up.

Guided Practice (Teacher and Student)	• Read aloud the question twice. • Guide student in using manipulatives and visuals • Student solves the problem with teacher guidance • Provide immediate affirmative and corrective feedback.	• Read aloud the question twice. • Guide student in using manipulatives and visuals • Apply the steps in the THINK framework and verbalizes reasoning related to the task. • Write the solution to the problem • Use probes and prompts to enhance student verbalizations • Ask appropriate questions to probe student thoughts if no verbalizations after 5 second • Provide immediate affirmative and corrective feedback.
Independent Practice (Student)	• Listen to teacher read aloud the question twice. • Solve word problems.	• Listen to teacher read aloud the question twice. • Use manipulatives and visuals. • Apply the steps in the THINK framework and verbalize reasoning related to the task. • Write the solution to the problem • Use probes and prompts to enhance student verbalizations • Ask appropriate questions to probe student thoughts if no verbalizations after 5 second
	• Teacher record student verbalizations. • Researcher transcribe and analyze student verbalizations. • Researcher assess student independent work using rubric.	• Teacher record student verbalizations. • Researcher transcribe and analyze student verbalizations. • Researcher assess student independent work using rubric.

Note. The instructional model represented in the table above incorporated the gradual release of responsibility. The teacher starts the lesson by (a) reviewing student's background knowledge and the expectations for the present lesson (I Do), and (b) provide guided instruction or practice (We Do). The student is then given an opportunity to practice the skills (You Do) (Fisher & Frey, 2008; Harlacher, Sanford, & Nelson, 2014; McCoy, 2011; Levy, 2007).

The verbalization lessons involved explicit guidance, teacher direction, guided instruction, and independent practice. The teachers (a) reviewed previous lessons and skills, (b) unpacked the present lesson objective(s), (c) independently applied and verbalized the steps in the THINK framework, (d) collaboratively practiced the new skills with the student by working through examples, (e) provided feedback about student work and responses, and (f) provided opportunity for the student to practice independently. As part of the explicit instruction model, the teachers were encouraged to verbalize their reasoning at an appropriate pace while using manipulatives, visuals, or equations to solve one-step word problems that consisted of a mixed set of addition, subtraction, multiplication, and division. Karp & Voltz (2000) noted that the use of explicit instruction for students with disabilities "uncover or make overt the covert thinking strategies that support mathematical problem solving. Students with disabilities may otherwise not have access to these strategies, as they may not autonomously acquire or apply them without explicit instruction." (p. 208).

As recommended by Montague (2008), the students were encouraged to verbalize their thinking processes and justify their answers out loud before writing them down. They used manipulatives, written sentences, pictures, or equations to show their work after verbalizing their thoughts to the teachers. The teachers used prompts and different questioning techniques to elicit student thinking. The teacher asked questions such as: What does the problem ask you to find? What important information did the problem give you? What strategy did you use (what did you do) to solve the problem? Does your answer make sense? Why or why not? (Thomas, 2006). This approach is similar to the "diagnostic interview" approach discussed by Harbour, Karp, and Lingo (2016). Diagnostic interviews involve teachers asking probing questions that explore the students' thinking regarding their misconceptions. During the lessons, teachers employed talk moves and other strategies from the professional development.

Treatment Fidelity

Process Evaluation RQ 1: To what extent did the teachers adhere to the verbalization intervention procedures?

As discussed in chapter 4, procedural/treatment fidelity is critical when implementing instructional interventions. Durlak and DuPre (2008) found that when interventions are implemented with high fidelity, they yielded effect sizes that are greater than interventions that were not implemented with fidelity. Two observations were conducted per teacher using the fidelity checklist (See Table 5). There were other low-stake observations to provide feedback to the teachers. A fidelity checklist was used to capture teacher adherence to the intervention procedures and quality of delivery (i.e., the manner in which a teacher delivers an intervention) (Dusenbury et al., 2003; O'Donnell, 2008; Nelson, Cordray, Hulleman, Darrow, & Sommer, 2012).

In single case experimental designs, adequate procedural fidelity must be at or above 80% (Horner et al., 2005). In the present study, inter-observer agreement (IOA) data and fidelity of implementation measures were obtained to determine whether the interventions were being implemented as intended. Two independent scorers (student researcher and mathematics specialist) completed the fidelity checklist during the observations. After each scorer completed the fidelity checks, the points were totaled, and data were compared to check for consistency. The fidelity of implementation was calculated as the number of correctly completed components or steps divided by the total number of steps required per lesson (in this case, 20 steps). Measured fidelity of implementation for both teachers reflected an averaged inter-rater score of 90% during the intervention phase (Teacher A, rater 1#, rater 2#; Teacher B, rater 1#, rater 2#).

Using the approach described by Horner et al. (2005) and McHugh (2012), inter-rater agreement on the observation checklists was calculated by dividing the number of agreements by the total number of agreements plus disagreements and multiplying the quotient by 100 (Agreement/ (Agreement + Disagreement) x 100). Inter-observer agreement was 95% for all teachers during the intervention phase. For single case experimental designs, the minimum standard for inter-observer agreement is 80% (Horner et al., 2005; McHugh, 2012).

Process Evaluation RQ 2: Did the students receive sufficient instruction and support to demonstrate mastery of the verbalization strategies? This question addresses the dosage received by the students. Three of the four students received all the prescribed dose of intervention. The fourth student (student B2) participated in 14/16 (88%) of the intervention sessions due to illness during the last week of school.

The students' problem-solving skills (i.e., conceptual understanding and procedural fluency) were evaluated using a rubric that measured their ability to understand, to plan, to solve, and to check each problem (see Table 14). The students earned scores that reflect their conceptual understanding (i.e., ability to understand the problem and show workable plan to solve the problem) and procedural fluency (i.e., correct implementation of the plan and ability to give a reasonable solution). For example, using the rubric below, if a student's response shows a partial understanding of the problem, he/she earned 2 points for understanding. Similarly, a student earned 4 points for providing responses that indicate a complete understanding of the problem.

Table 14

Problem-solving scoring rubric

	4 points	3 points	2 points	1 point
Understand	Response indicates insight and complete understanding of the problem	Response indicates understanding of the problem	Response indicates partial understanding of the problem	Response indicates misunderstanding of the problem
Plan	Makes original/ creative plan to solve the problem	Shows workable plan to solve the problem	Shows a plan that will not solve the problem	Produces unworkable plan Does not organize data Chooses no strategy or chooses an incorrect strategy
	Organizes data concisely and with insight	Organizes data appropriately	Partially organizes data	
	Uses one or more strategies to solve problem	Chooses a strategy to solve the problem	Chooses an inappropriate strategy to solve the problem	
Solve	Shows clear, well-organized implementation of the plan	Shows correct implementation of the plan	Shows partially correct implementation of the plan	Shows incorrect implementation of the plan
	Clearly shows	Shows some evidence of the	Shows little	Produces work that is unrelated to the

	logical processes used in implementation Uses data that fit the information given in the problem	processes used Makes few or no errors in data	evidence of the processes used Produces work having many errors	problem
Check	Attains clear, reasonable solution that is meaningful to the problem Clearly labels all parts Gives clear, insightful reasons to explain the accuracy of the solution If solution is not reasonable, shows evidence of choosing another strategy	Finds reasonable, acceptable solution Labels most parts Gives an explanation for the solution If solution is not reasonable, shows some evidence of redoing the problem	Produces partially acceptable solution Labels no parts or few parts Gives incomplete or unclear explanation for the solution If solution is not reasonable, shows no evidence of redoing the problem	Attains unreasonable solution or a solution that is unrelated to the problem Uses no labels Gives no explanation for the solution

Note. This rubric was developed by Thomas (2006).

Statistical and Visual Analysis

As discussed in chapter 4, visual inference of graphed data remains the standard by which single case experimental design (SCED) data are most commonly analyzed (Borckardt et al., 2008; Brossart et al., 2006; Parker, Cryer, & Byrns, 2006; Kratochwill et al., 2010, 2013; Lane & Gast, 2013). “Visual analysis of graphic displays of data is a cornerstone of studies using a single case experimental design (SCED).” (Lane & Gast, 2013, p. 1). Following this further, “top experts in single-case analysis champion the use of statistical methods alongside visual analysis whenever it is appropriate to do so.” (Smith, 2013, p.13). Therefore, for the present study, statistical methods and visual analysis of data were conducted. Guided by the literature on determining the effects of the verbalization interventions (Lane & Gast, 2013), the following features were examined: level, stability, variability, and trends across baseline and intervention phases.

In order to determine the level or stability of data in SCED studies, researchers often calculate the median for the dependent variable for each phase for each student/intervention recipient. If 80% of a student's data fell within 20% of the median data can be considered stable (Gast & Spriggs, 2010). Also, trend line can either be drawn by hand or created using a computer program such as Microsoft Excel (McDougal, Graney, Wright, & Ardoin, 2010). Drawing a trendline by hand is usually accomplished using the split-middle method: identify the middle data point for each phase, (b) calculate the mid-rate and mid-date, and (c) draw a line between mid-rate and mid-date for both baseline and intervention phases to determine whether the line is accelerating, decelerating, or zero-celerating (Gast & Spriggs, 2010). On the other hand, the Microsoft Excel program uses the ordinary least squares regression to summarize the data and then plots the line accordingly. The present study employs the trendline created by the Microsoft Excel spreadsheet software for more accuracy.

Determining effect sizes using the statistical analysis may include the utilization of the standardized mean difference (SMD), percentage of non-overlapping data (PND), and Mean Baseline Reduction methods (MBR) (Olive & Smith, 2005; Smith, 2013). During a personal communication about the present study (May 2016) Dr. Kratochwill, a leading scholar and textbook author in SCED research recommended the standardized mean difference (SMD) method. This is consistent with the established guidelines from Olive and Smith (2010) who compared five alternative methods for assessing the magnitude of effect sizes for SCED studies and recommended the SMD approach.

The SMD is calculated by subtracting the mean of the baseline phase from the mean of the intervention phase, then dividing this answer by the standard deviation of baseline (Bray &

Kehle, 2013; Hedges, Pustejovsky, & Shadish, 2014). SMD could be calculated in two ways: SMDall and SMD3. In SMDall, all the baseline and intervention data points are used. While in SMD3, only the last three data points of baseline and intervention phases are used. Some researchers have argued that using only the last three data points of baseline and intervention may increase the effect size because the last few sessions are usually the best. Therefore, SMD3 results are often considered inflated. Based on these concerns, the SMDall approach is used to calculate the magnitude of the verbalization intervention effect. SMDall is the standard for calculating effect sizes in between-subjects designs, and it is mathematically equivalent to the Cohen's d (Hedges, Pustejovsky, & Shadish, 2012; Shadish, 2012).

Cohen (1988) cautiously define effect sizes as "small, d = .2," "medium, d = .5," and "large, d = .8". These values should be used as a general rule of thumb (Durlack, 2009; Hedges & Hedberg, 2007). Durlak (2009) argues that 'assuming that "large" effects are always more important than "small" or "medium" ones is unjustified.' (p. 923). Several educational researchers and psychologists have established that effects sizes around 0.20 are of policy interest, especially when they are based on measures of student achievement (Hedges & Hedberg, 2007).

Results

Student A1 results. Student A1 is a 9 year and 6-month-old African-American girl. she has been receiving special education services for more than five years. Based on the most recent comprehensive psychoeducational evaluation conducted in December 2016, she earned a full-scale IQ (FSIQ) score of 73 (4th percentile) on the Wechsler Intelligence Scales for Children, Fifth Edition (WISC-V). Her Verbal Comprehension Index was measured in the extremely low range and is an area of relative weakness (SS=68). Based on this score, it is expected that the student will struggle considerably to keep up with same age peers in subjects that rely heavily on reading and language. Additionally, she earned a very low score on the Fluid Reasoning Index (FRI; SS=79). This score suggests that she struggles with identifying underlying conceptual rules, as well as understanding quantitative concepts of equality.

Figures 13, 14 and 15 illustrate the performance of Student A1 from the baseline through the intervention phase. The student participated in eighteen sessions (six baseline and twelve intervention). The verbalization intervention was introduced after a stable or predictable pattern of performance was established during the baseline phase. The stable baseline allows stronger causal inference to be drawn (Gast, 2013; Gast & Spriggs, 2010; Lane and Gast, 2013). Stability refers to a lack of slope, and low variability during the baseline phase. Using the rubric presented in Table 14, the student scored an average of 1.08 out of 4.00 during the baseline phase.

The student work samples and observations revealed the following behaviors/skills during the baseline phase: (a) misunderstanding of the problem (especially for subtraction, multiplication and division word problems); (b) selection of inappropriate strategy; (c) incorrect implementation of the plan; and (d) inability to give appropriate or reasonable explanations for the solution. The student, who was in fourth grade at the time of the study visibly struggled while attempting to solve the 2nd/3rd-grade word problems. As has been found typical in struggling students (Kingsdorf & Krawec, 2014; Schumacher and Fuchs, 2012), this student demonstrated a tendency to attempt to solve all the problems by adding all numbers in the word problem. This implies and may be related to lack of understanding of the problem structure. To help address such issues, Van de Walle, Karp, Lovin, and Bay-Williams (2014) suggest that teachers should provide explicit instruction about different problem structures. Recognizing the structure or characteristics of a word problem can then be leveraged to help students determine which operation(s) to use.

Based on observations data, the student was found to be actively engaged throughout the lessons. During the intervention sessions, she received corrective feedback about her work. For conceptual understanding and procedural fluency, she scored an average of 2.10 and 1.80 respectively leading to an average of 1.96 out of 4.00 during the intervention phase. These scores suggest that the verbalization intervention had a greater impact on the student's conceptual understanding than the procedural fluency. The visual analysis indicates an upward trend for both the conceptual understanding and procedural fluency.

The change in data variability or bounce (ranges from 1-3) during the intervention phase might be systematically related to the problem structure. For example, the student tended to score between 2.00 and 3.00 if the problems involved "Result Unknown" or "Unknown Product." On the other hand, the student tends to score between 1.00 and 2.00 if the problems involve "Change Unknown," "Start Unknown," "Group Size Unknown," or "Number of Groups Unknown." See Tables 5.3 and 5.4 for specific examples of these problem structures. The SMDall or Cohen's d for student A1 is: Mean (intervention) – Mean (baseline) / Pooled SD (intervention + baseline). Cohen's d = (1.96 - 1.08) / (0.94 + 0.20) = 0.88 / 1.14 = 0.77. The SMDall or Cohen's d is 0.77, suggesting that thinking aloud while solving mathematical problems was an effective (medium) intervention for student A1 (Cohen, 1988).

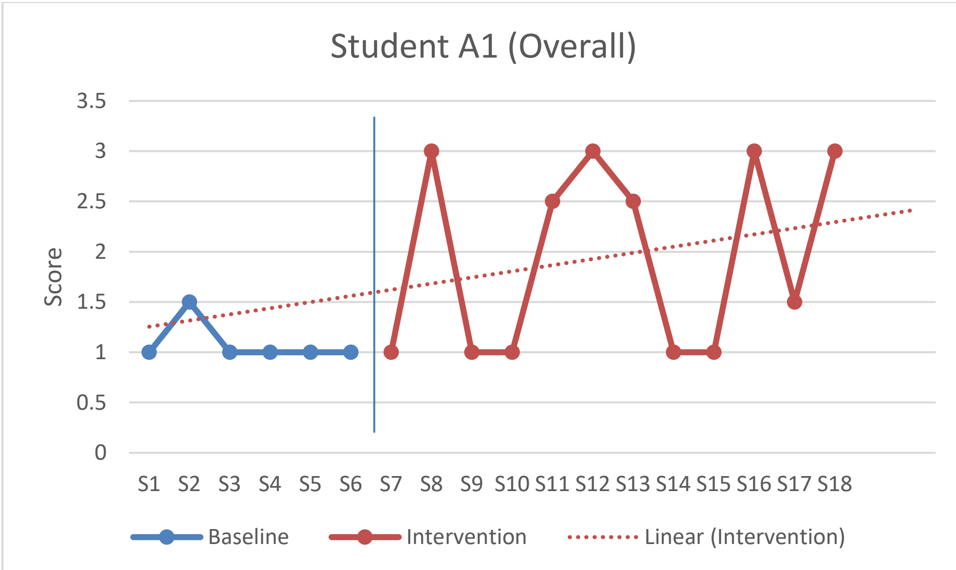


Figure 13. This figure shows student A1 overall performance.

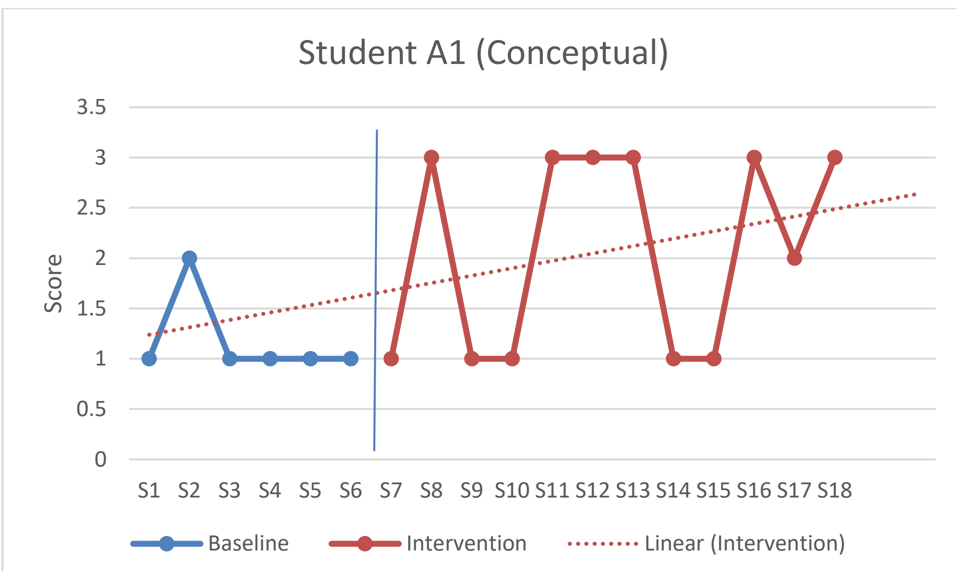


Figure 14. This figure shows student A1 conceptual understanding.

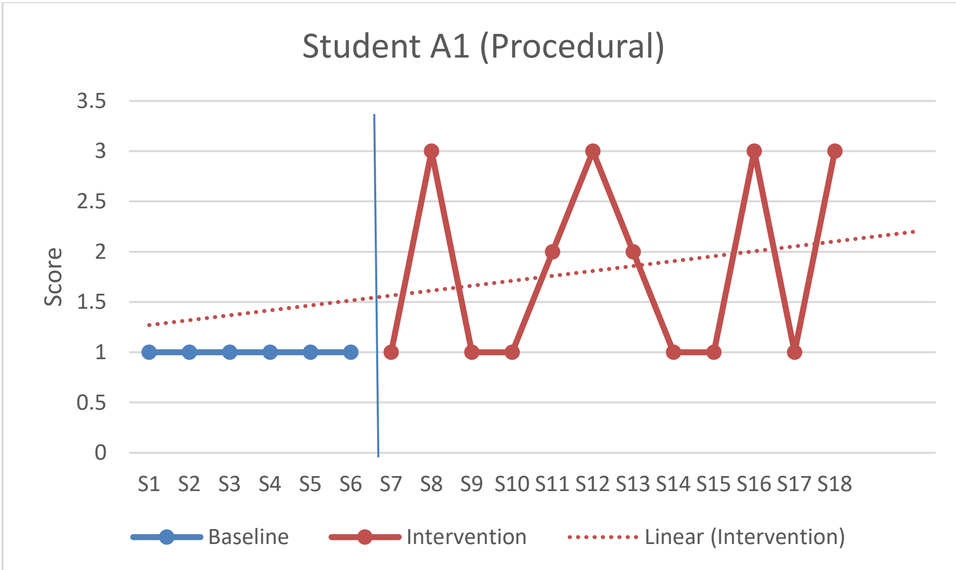


Figure 15. This figure shows student A1 procedural fluency.

Student A2 results. Student A2 is a 10-years and 4-months-old Hispanic-American girl who was referred for a psychoeducational evaluation by her teachers in 3rd grade due to academic concerns. Based on the results of the comprehensive psychoeducational evaluation conducted in November 2015, her general cognitive ability is within the low average range. Her overall thinking and reasoning abilities exceed approximately 27% of her peers. Her Cognitive Proficiency Index (CPI = 72), as demonstrated by her performance on working memory and processing speed tasks, fell in the below average range.

Figures 16, 17 and 18 illustrate the performance of Student A2 from the baseline through the intervention phase. The student participated in twenty-six sessions (ten baseline and sixteen intervention). The verbalization intervention was introduced after a stable or predictable pattern of performance was established during the baseline phase. The student scored an average of 1.20 out of 4.00 during the baseline phase. Student work samples and observations revealed that the student demonstrated specific skill deficits such as (a) inability to correctly identify operations needed to solve the problem; (b) inability to set up the problem's manipulatives; (c) difficulty remembering math facts; and (d) inability to solve problems quickly and efficiently.

Further, the student gave an incomplete or unclear explanation for the solution and sometimes produced work that was unrelated to the problem. The errors and misconceptions shown by student A2 are similar to those discussed by Kingsdorf and Krawec (2014): errors in the categories of number selection, operation selection, omissions and mistakes stemming from lack of self-monitoring. Paré-Blagoev and colleagues (2014) recommend that determining which errors are most persistent in student work (for example, in Algebra I) can help "focus the attention of both researchers and practitioners towards developing and utilizing interventions to remediate misconceptions at the most critical and effective times" (p. 11). Similar to prior findings regarding the errors exhibited by students who struggle with mathematics, student A2 often used addition and subtraction operations to solve all the problems during the baseline phase (Kingsdorf & Krawec, 2014; Schumacher and Fuchs, 2012).

The student experienced an immediate positive effect after two intervention sessions were introduced and completed. The student scored an average of 2.55 out of 4.00 during the intervention phase. She scored an average of 2.72 and 2.38 in the areas of conceptual understanding and procedural fluency respectively. Given the lack of verbalization instructions during baseline phase, the jump from baseline mean (1.20) to intervention mean (2.55) may be associated with the instructional intervention provided by teacher A. Similar to student A1, the conceptual understanding and procedural fluency scores suggest that the verbalization intervention had a higher impact on the student's conceptual understanding than the procedural fluency. The visual analysis indicates an upward trend for both the conceptual understanding and procedural fluency.

The SMDall or Cohen's d for student A2 is: Mean (intervention) – Mean (baseline) / Pooled SD (intervention + baseline). Cohen's d = (2.55-1.20) / (0.73+0.26) = 1.35 / 0.99 = 1.36. The SMDall or Cohen's d is 1.36, suggesting that thinking aloud while solving mathematical problems was an effective (large) intervention for student A2 (Cohen, 1988).

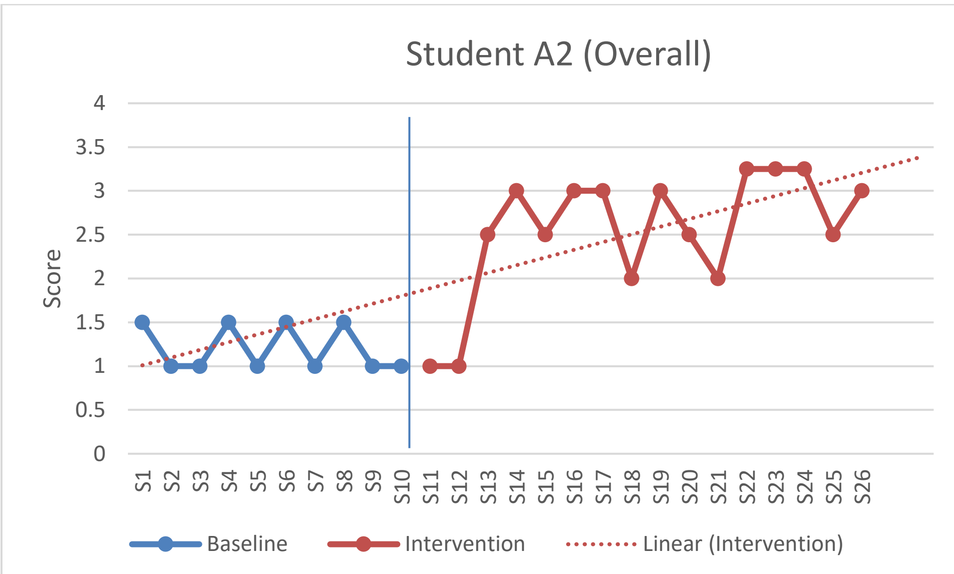


Figure 16. This figure shows student A2 overall performance.

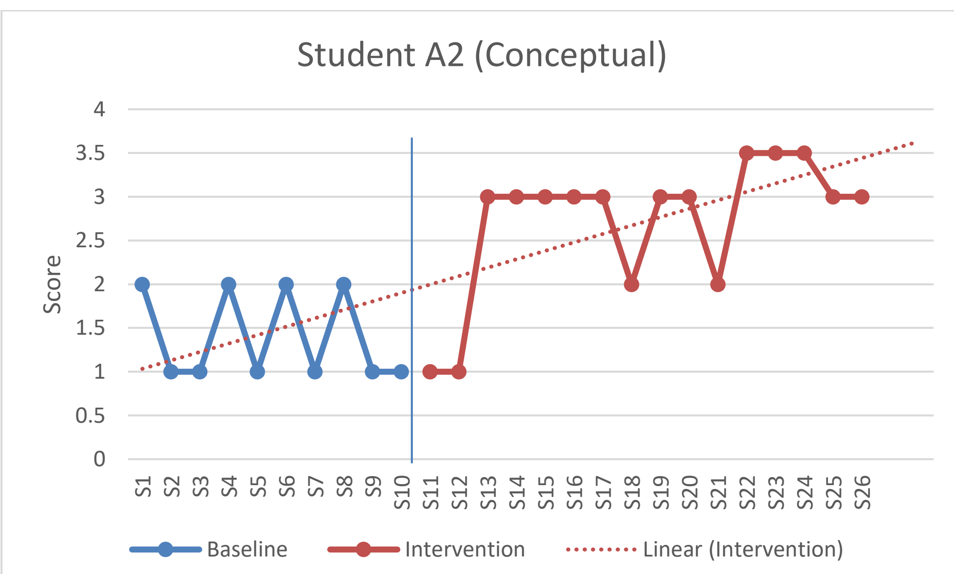


Figure 17. This figure shows student A2 conceptual understanding.

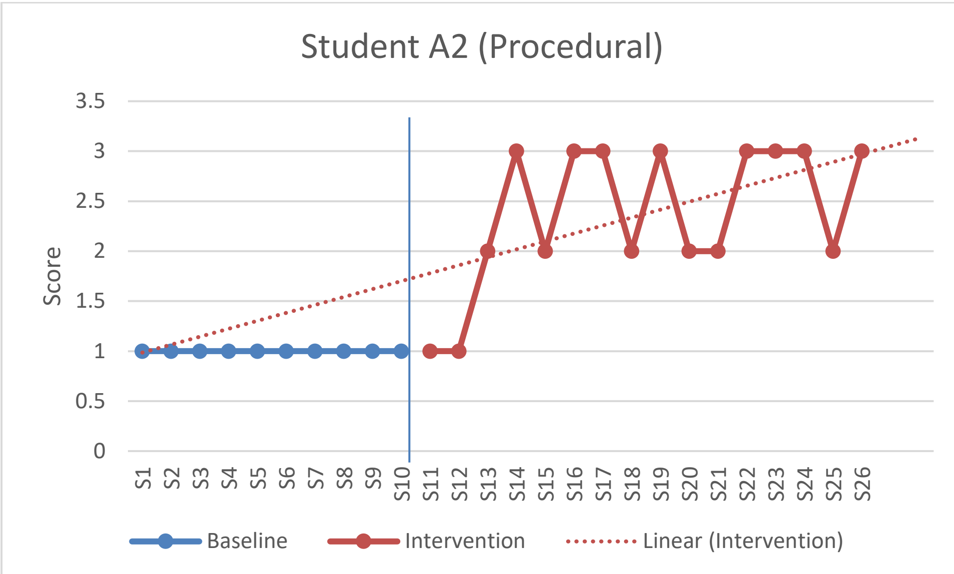


Figure 18. This figure shows student A2 procedural fluency.

Student B1 results. Student B1 is a 10-year and 8-month-old African-American male who has been receiving special education and related services for more than three years. Based on the results of the comprehensive psychoeducational evaluation conducted in December 2014, his general cognitive ability, as estimated by the Wechsler Intelligence Scale for Children, 4th Edition (WISC-IV), was in the low average range of intellectual functioning. His overall thinking and reasoning abilities exceed those of approximately 10% of children his age (FSIQ = 81 = 10th percentile). The student performed much better on verbal than on nonverbal reasoning tasks.

His reasoning abilities and concept formation were in the Low Average range and above those of only 19% of his peers (VCI = 87 = 19th percentile). His general perceptual/nonverbal reasoning abilities were above those of 3% of his peers (PRI = 71 = 3rd percentile). The student's ability to sustain attention, concentrate, and exert mental control is in the average range. His Working Memory Index was better than 27% of his peers. Finally, student B1 performed better than approximately 34% of his peers on the processing speed tasks (Processing Speed Index = 94 = 34th percentile).

Figures 19, 20 and 21 represent the performance of student B1 during the baseline and intervention phases. He participated in eighteen sessions (i.e., six baseline and twelve intervention sessions). The student scored an average of 1.08 out of 4.00 during the baseline phase. During the baseline period, his verbal and written responses indicated a misunderstanding of word problems. The student had difficulty remembering math facts and struggled to solve problems quickly and efficiently. Based on the observations conducted during the baseline sessions, the student frequently relied on keyword/phrase strategy to solve the problems Karp, Bush, & Dougherty (2015) assert that solely depending on keyword strategy "removes the act of making sense of the actual problem from the process of solving word problems." (p. 212). The researchers noted that students who rely on keywords often "overgeneralize by stripping numbers from the problem and using them to perform a computation outside the problem context." (p. 212). Consistent with this finding in the literature, while solving a word problem, student B1 thought that the word “left” always implies subtraction. The teacher clarified that word problems sometimes involve keywords or phrases that are contrary to the meaning of the problem. Case in point: John took 10 crayons he no longer wanted and gave them to Mary. Now John has 5 crayons left. How many crayons did John have to begin with? In this situation, the students subtracted 5 crayons from 10 crayons, based on the keyword strategy.

During the intervention phase, the teacher modeled and verbalized her mathematical reasoning and solutions to problems while exploring different addition, subtraction, multiplication, and division problems. Because the student was used to searching for keywords, it was necessary for the teacher to model how to make sense of the problem. The student used manipulatives such as snap cubes and ten frames and created drawings of manipulatives to show his thinking. He did not use number lines to solve the problem, even though the teacher used number lines during the lessons.

The student received the intervention for twelve sessions and scored an average of 2.71 out of 4.00. For conceptual understanding and procedural fluency, the student scored an average of 2.88 and 2.54 respectively. These scores suggest that the verbalization intervention had a greater impact on the student's conceptual understanding than the procedural fluency. The visual analysis indicates an upward trend for both the conceptual understanding and procedural fluency. The SMDall or Cohen's d for student B1 is: Mean (intervention) – Mean (baseline) / Pooled SD (intervention + baseline). Cohen's d = (2.71 - 1.08)/ (0.74 + 0.20) = 1.63 / 0.94 = 1.73. The SMDall or Cohen's d is 1.73, suggesting that thinking aloud while solving mathematical problems was an effective (large) intervention for student B1 (Cohen, 1988).

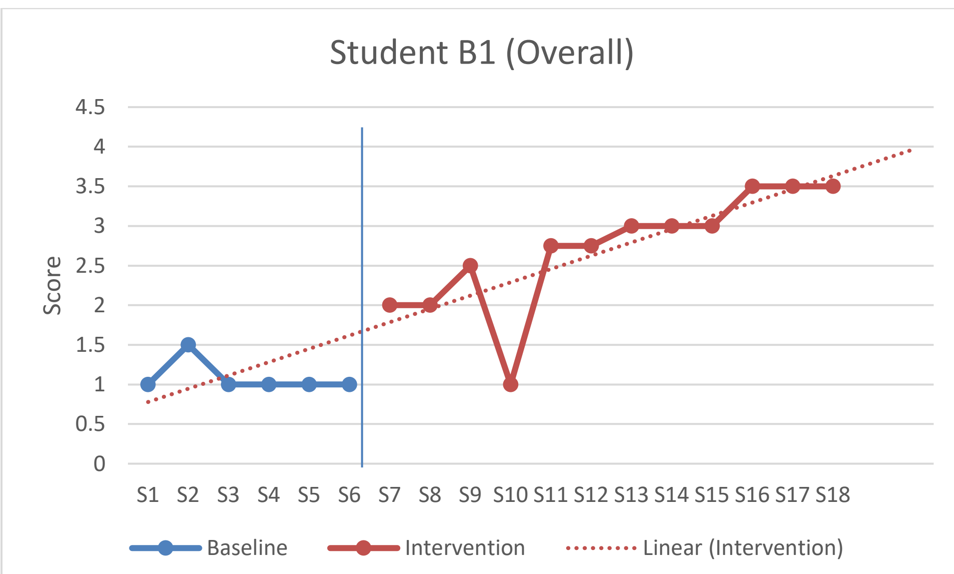


Figure 19. This figure shows student B1 overall performance.

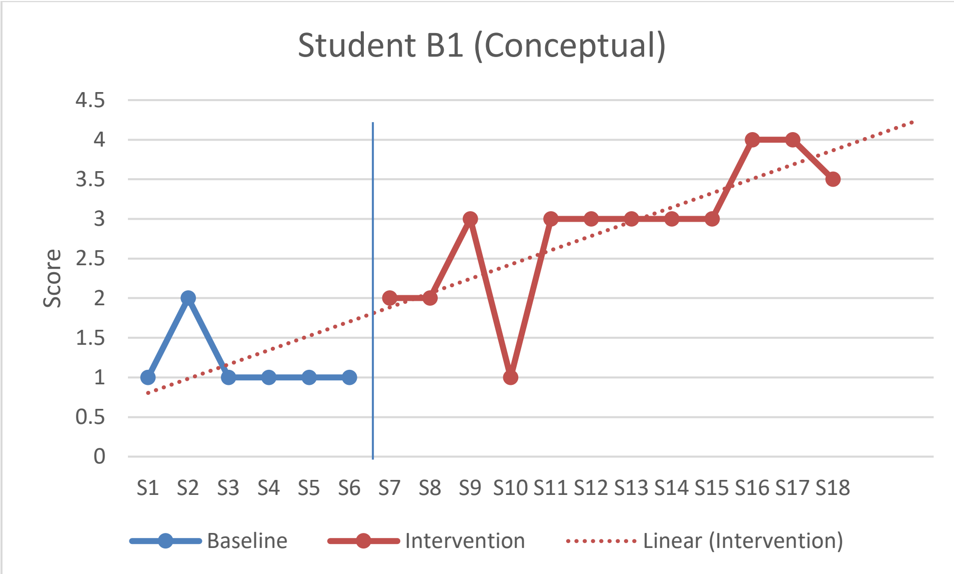


Figure 20. This figure shows student B1 conceptual understanding.

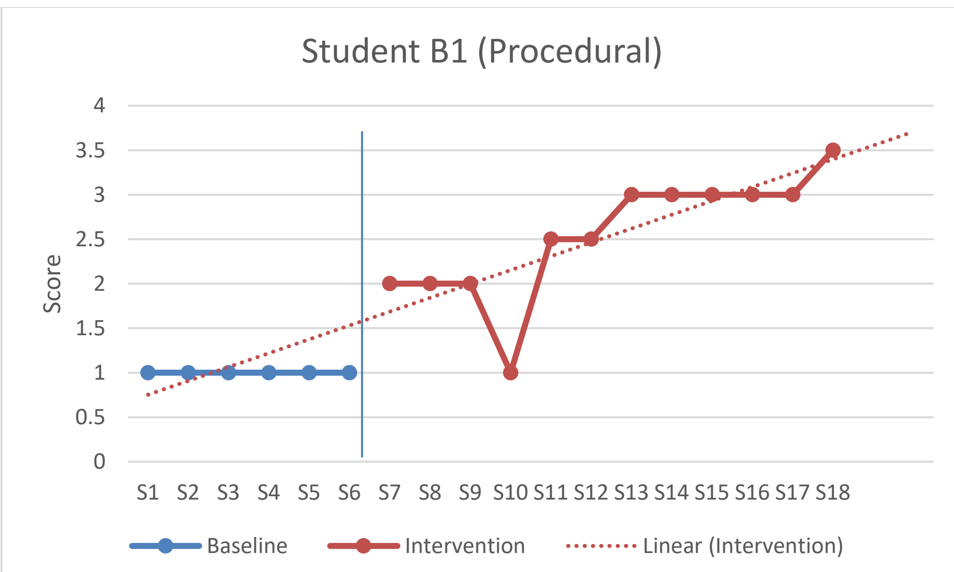


Figure 21. This figure shows student B1 procedural fluency.

Student B2 results. Student B2 is a 10-year-old Hispanic-American girl. She has been receiving special education and related services since May 2015. Based on the results of the comprehensive psychoeducational evaluation conducted in April 2015, the student's cognitive ability, as estimated by the Wechsler Intelligence Scale for Children, 4th Edition (WISC-IV), was in the Borderline range. Her overall thinking and reasoning abilities exceed those of 4% of children her age (FSIQ = 74 = 4th percentile). The student's verbal comprehension (VCI) and perceptual reasoning (PRI) abilities were both in the low average range (VCI = 87 = 19th percentile, PRI = 84 = 14th percentile). Her general working memory abilities were in the borderline range (WMI = 77 = 6th percentile), and general processing speed abilities in the extremely low range (PSI = 68 = 2nd percentile).

Figures 22, 23, and 24 illustrate the performance of student B2 during the baseline and intervention phases. She participated in twenty-four sessions (i.e., ten baseline and fourteen intervention sessions). A predictable pattern of performance was established during the ten baseline sessions. The student scored an average of 1.60 out of 4.00 during the baseline phase, and demonstrated the following behaviors: (a) struggled to make sense of the word problems, (b) made computational errors such as regrouping incorrectly, (c) required extended time to complete the word problems; and (d) used inefficiently strategies. This student persevered the most among all the intervention recipients. She took additional time to complete the task and showed her work clearly, either accurate or inaccurate. The classroom teachers confirmed this behavior in the classroom as well. The baseline scores range from 1.00 to 2.50.

During the intervention phase, teacher B taught student B2 how to verbalize her reasoning while solving mixed sets of addition, subtraction, multiplication, and division word problems using the THINK framework. The teacher modeled with multiple representations, supported the student to use manipulatives (e.g., snap cubes, ten frames), and visually represented her thinking while solving the word problems. The student also received corrective feedback about her work from the teacher. The teacher ensured that the lessons addressed the misconceptions and data identified during the prior lesson.

The student received the intervention for sixteen sessions and scored an average of 2.86 out of 4.00. For conceptual understanding and procedural fluency, the student scored an average of 2.93 and 2.68 respectively. These scores suggest that the verbalization intervention had a greater impact on the student's conceptual understanding than the procedural fluency. The visual analysis indicates an upward trend for both the conceptual understanding and procedural fluency. The SMDall or Cohen's d for student B2 is: Mean (intervention) – Mean (baseline) / Pooled SD (intervention + baseline). Cohen's d = (2.86 - 1.60)/ (0.78 + 0.77) = 1.26 / 1.55 = 0.81. The SMDall or Cohen's d is 0.81, suggesting that thinking aloud while solving mathematical problems was an effective (large) intervention for student B2 (Cohen, 1988).

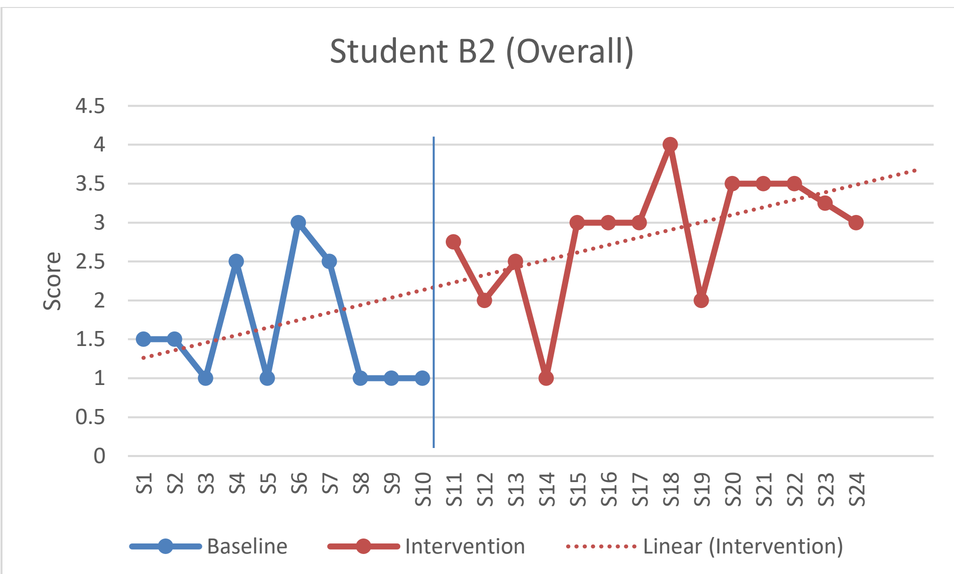


Figure 22. This figure shows student B2 overall performance.

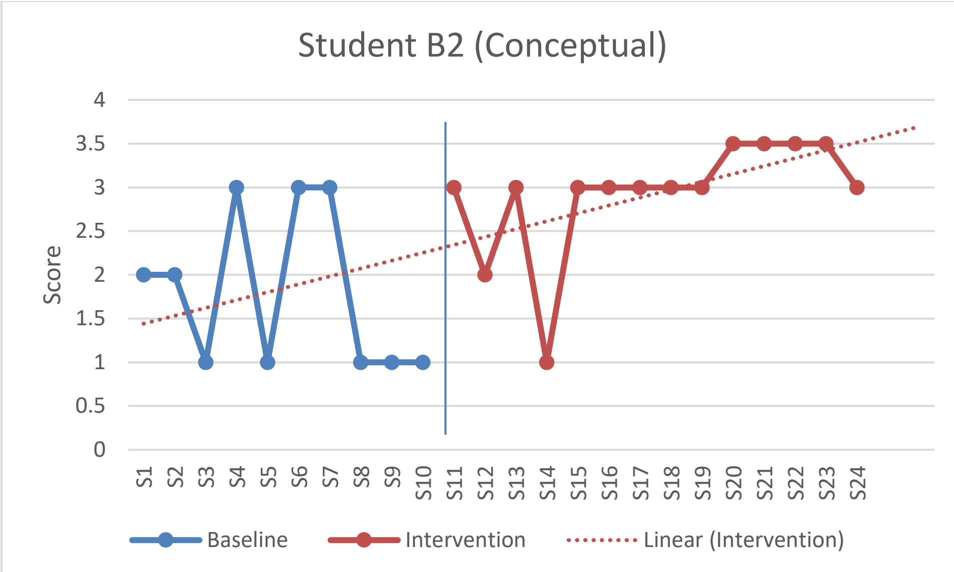


Figure 23. This figure shows student B2 conceptual understanding.

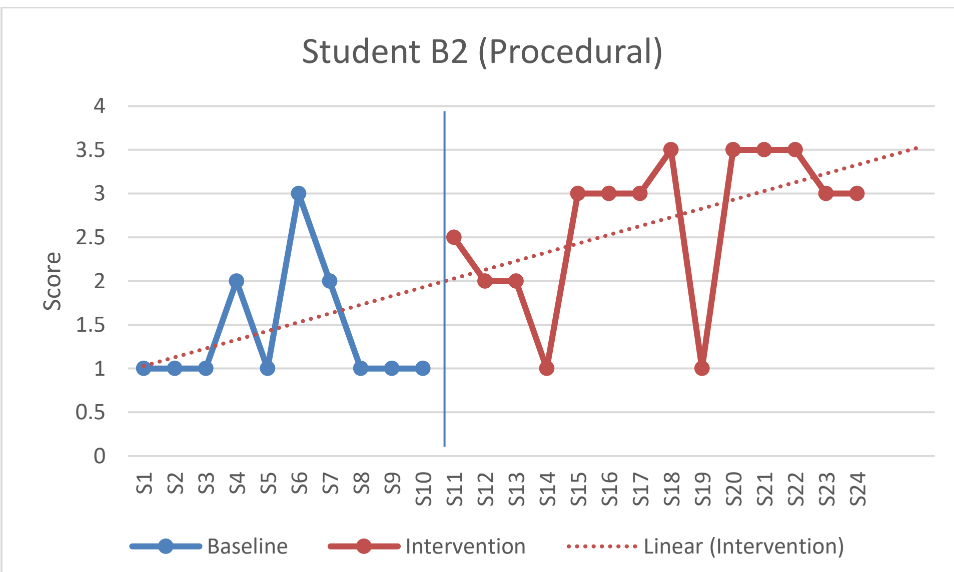


Figure 24. This figure shows student B2 procedural fluency.

Discussion

In relation to the outcome evaluation, the present study sought to answer the following overarching research question: To what extent does the verbalization intervention affect the mathematical problem solving (conceptual understanding and procedural fluency in solving word problems) of fourth-grade students with MLD? Table 15 gives an overview of the study results.

Table 15

Overview of the study results

	# of session	Baseline Phase: Overall Performance (Mean)	Intervention Phase: Overall Performance (Mean)	Intervention Phase: Conceptual Understanding (Mean)	Intervention Phase: Procedural Fluency (Mean)	Cohen's d effect size
Student A1	6 baseline and 12 intervention	1.08	1.96	2.10	1.80	0.77
Student A2	10 baseline and 16 intervention	1.20	2.55	2.72	2.38	1.36
Student B1	6 baseline and 12 intervention	1.08	2.71	2.88	2.54	1.73
Student B2	10 baseline and 14 intervention	1.60	2.86	2.93	2.68	0.81
Average	8 baseline and 13.5 intervention	1.24	2.52	2.66	2.35	1.17

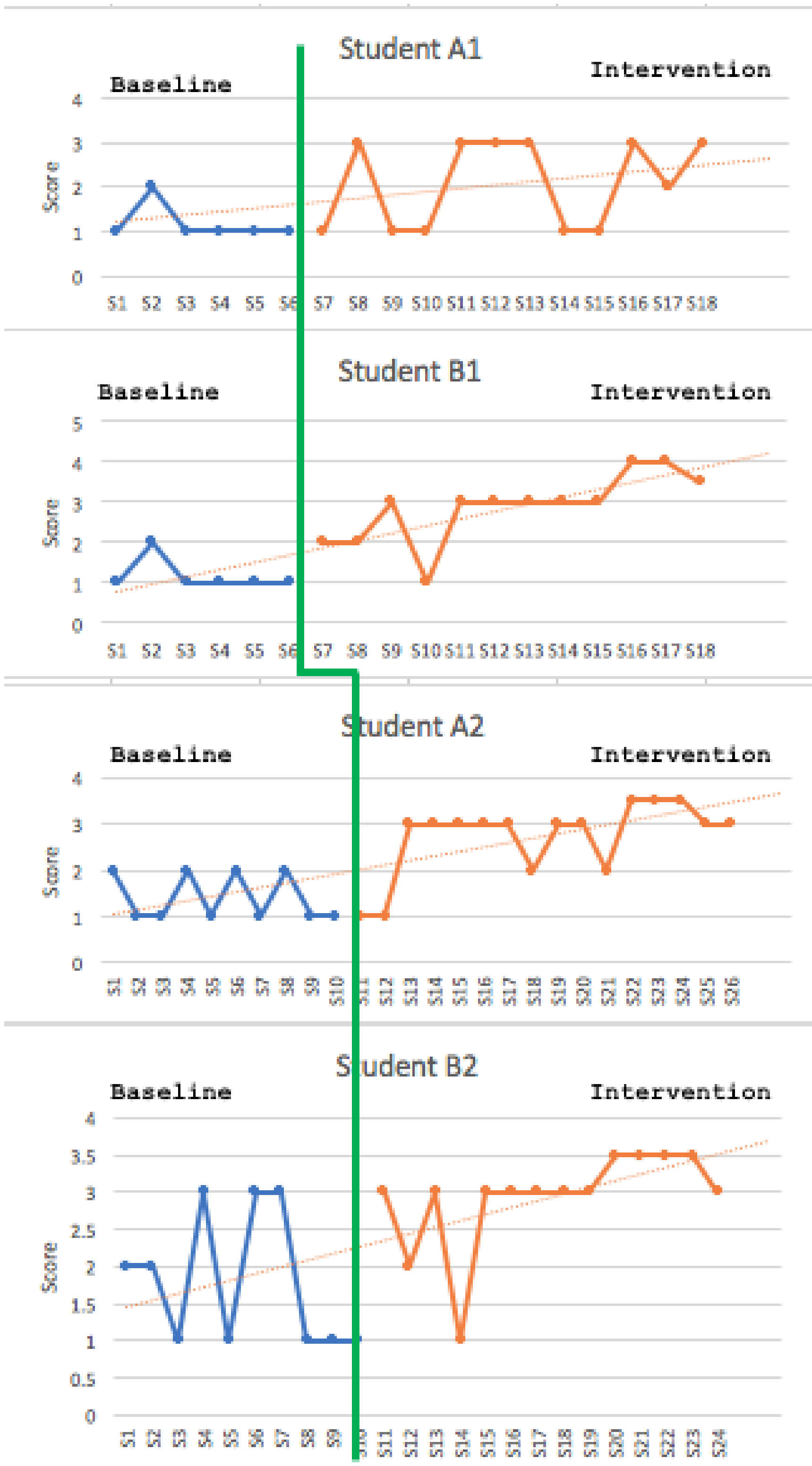


Figure 25. Conceptual understanding graphs for all students.

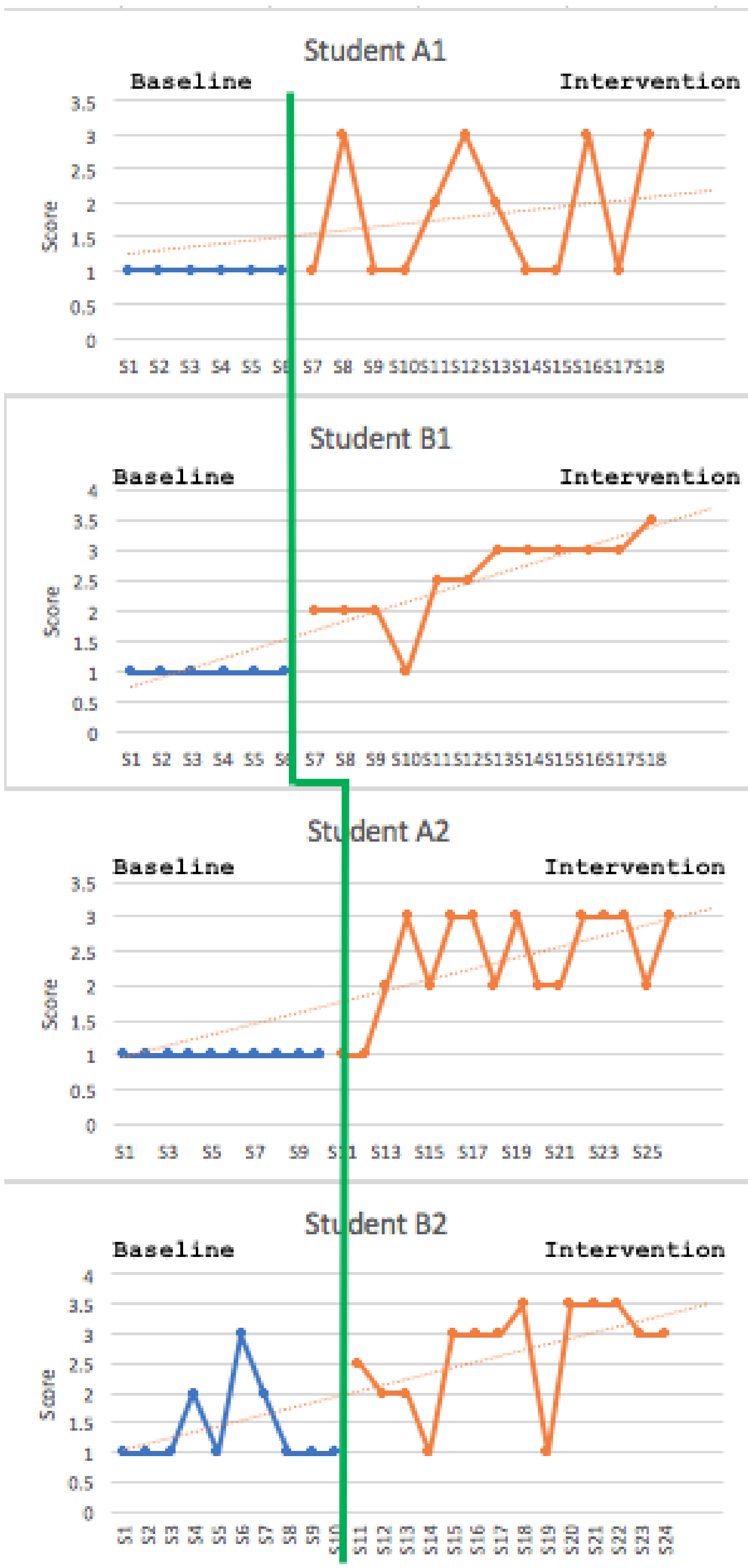


Figure 26. Procedural fluency graphs for all students.

The mean Cohen's d effect size of all the intervention recipients was 1.17 (range from 0.77 to 1.73), suggesting a large effect when students with MLD receive explicit instruction on how to verbalize their thinking while solving mathematical problems. This value is consistent with the effect sizes of prior RCT and SCED studies that focused on mathematics intervention for students with learning disabilities (Baker et al., 2002; Gersten et al., 2009; Jayanthi et al., 2008; National Mathematics Advisory Panel, 2008). For example, Gersten and colleagues (2009) analyzed eight studies in the area of student verbalizations (7 RCTs and 1 QED) and reported a mean effect size of 1.04. Previous research has found that students with MLD can develop both conceptual understanding and procedural fluency when adequately taught and given the opportunity to verbalize the reasoning while solving word problems (Baker et al., 2002; Jayanthi, Gersten, & Baker, 2008; Karp & Volt, 2000).

Baseline data established that before the introduction of the verbalization interventions the students presented with difficulties in both conceptual and procedural knowledge of different mathematical concepts. Specifically, all the students struggled to represent and solve addition, subtraction, multiplication, and division word problems selected from common core standard two years below their current grade level. Consistent with previous research on the effectiveness of student verbalizations, the students demonstrated a change in mathematical behaviors from baseline to intervention. As student performance during baseline showed a mean score of 1.24 and at intervention the mean score was 2.52, it is believed that the verbalization interventions had a positive impact on student achievement. The gains experienced by the students may be associated with intervention having provided students with multiple opportunities to verbally plan and process the word problems.

Although all the students experienced gains in their mathematical problem-solving abilities, the underlying reasons for the variation in the effectiveness of the verbalization intervention among them are still unclear. The effect sizes, ranging from 0.77 to 1.73, might be related to student and teacher characteristics (e.g., gender, socioeconomic status, place of residence). At this time, there is no consensus among researchers regarding the relationship between student characteristics (e.g., age, inequity variables) and effect sizes. For example, Gersten et al., (2009) and Shalev (2004) imply that younger children benefit more from mathematics interventions than older children. Also, Siegler and Ramani (2008) and Wilson, Dehaene, Dubois, and Fayol (2009) have associated inequity variables such as low socioeconomic status with poorer outcomes from mathematics interventions. It is important to remind the reader that all the students that participated in the present study are from low socioeconomic status households.

Because the multiple baseline approach often involves staggered start points, the duration of the intervention may also play a role in the variability of outcomes. There have been conflicting reports about the relationship between effect sizes and mathematics intervention duration. Some researchers found that shorter mathematics interventions were more effective (Gersten 2009; Kroesbergen & Van Luit, 2003). A possible justification is that "short interventions tend to focus on a very small and specific domain of knowledge, such as addition up to 10." (Kroesbergen & Van Luit, 2003, p. 110). Conversely, Fischer, Moeller, Cress, and Nuerk (2013) found no relationship between effect size and mathematics intervention duration; while the Codding, Burns, and Lukito (2011) found that longer interventions (30 or more sessions) yielded higher effect sizes. It is possible that longer duration might allow more or all students to reach the maximum potential benefits from this intervention.

Implications for Practice

This research confirms previous findings and contributes to our understanding of effective instructional practices for teaching mathematics to elementary students with learning disabilities. The study holds implications for practice as it affirms the need for educators to model and give students ample opportunities to think aloud or verbalize the decisions they make while solving problems. The core instructional practices associated with the verbalization intervention (as used in the present study) include: (a) teachers modeling of their thinking, talking through the steps and clarifying the reasons for each step; (b) effective use of cognitive and metacognitive problem-solving strategies such as the THINK framework; (c) providing students with a set of appropriate questions and prompts to elicit their thinking; (d) encouraging students to use multiple representations (e.g., pictures or diagrams, equations, expressions, manipulatives) while solving word problems; (e) providing guided practice and corrective feedback; (f) reading the word problem to students if they are struggling readers; and (h) discouraging the students from using the keyword strategy. The combined effect of these practices is expected to yield a positive outcome for students with MLD.

Explicit modeling. Teachers should explicitly model their mathematical reasoning, explaining the steps and clarifying the reasons for each step. While describing the mathematics instructional framework for facilitating the inclusion of students with disabilities into general education classes, Karp and Voltz (2000) clarify that teacher modeling should involve demonstrating the steps to accomplishing a task as well as verbalizing the thinking process and reasoning that accompanies the steps. Students with learning disabilities can internalize different cognitive strategies when the teachers explicitly model the strategies (Montague, 2003).

Explicit modeling of the strategies is critical because “students with LD may not have the metacognitive resources compared with their higher ability peers and may actually shut down cognitively when confronted with problems that are difficult or that they perceive as difficult.” (Rosenzweig et al., 2011, p. 550). Further, Rosenzweig and colleagues (2011) found that verbalizations helped students with MLD internalize metacognitive skills and resources (e.g., ability to self-monitor, self-instruct, self-question, and self-correct statements/questions) required to solve mathematical problems. Student use of cognitive and metacognitive strategies become automatic during problem-solving through practice and with teacher feedback (Hutchinson, 1993; Montague, 2003; Naglieri & Johnson, 2000; Rosenzweig et al., 2011).

Questioning. The impact of effective questioning techniques on student verbalizations cannot be overemphasized. Prior mathematics intervention studies that deemphasized teacher questioning and feedback reported minimal effect sizes. For example, Schunk and Cox (1986) investigated the impact of verbalizations on the mathematics performance of students with MLD. The researchers instructed students to verbalize their thoughts while solving problems without providing the students with a set of questions or prompts. The students only received "effort-attributional feedback" to encourage them to persevere at tasks (e.g., "That's good. You're really working hard.") (p. 208). Their approach resulted in the smallest effect size (0.07) among the verbalization intervention studies reported by Gersten and colleagues (2008).

Taking into account the importance of effective questioning on student verbalizations, the present study involved pre-intervention teacher training that highlighted the types of questions and effective use of questioning techniques. Teachers learned five different kinds of questioning techniques that can be used to foster algebraic thinking: managing, clarifying, orienting, prompting mathematical reflection, and eliciting algebraic thinking (Driscoll, 1999). During the intervention sessions, the teachers used these questioning techniques to help the students explain and clarify their thinking. Similarly, the fourth-grade students were taught to ask themselves questions as they solve the word problems. Other related practices are using prompt cards and sentence starters. For older students who can read the word problems, prompt cards for self-questioning are suggested (Hutchinson, 1993). Also, Carr and Bertrando (2012) emphasize the importance of sentence starters for students who struggle to verbalize their thoughts. The present study involved the use of sentence starters to foster student verbalizations. If required the teachers used the sentence starters such as, “the answer makes sense because _______.”

Multiple representations. Van de Walle, Karp, and Bay-Williams (2010) advise that “it is sometimes difficult for students (of all ages) to think about and test abstract relationships using only words or symbols.” (p. 27). Along similar lines, Harbour, Karp, and Lingo (2016) affirm that "through the interwoven use of concrete materials, other visual representations (e.g., drawings), and numerical representations during instruction, students are able to manipulate, model, and symbolize mathematical concepts, which allows students to develop a deeper understanding of the content" (p. 131). In this regard, the authors agreed with Lesh, Post, and Behr’s (1987) recommendation that mathematical concepts should be presented to students in five different representations: manipulative models, real-world situations, oral language, written symbols, and pictures.

The use of multiple representations is particularly beneficial for promoting algebraic thinking in young children. While discussing the prerequisite algebra skills and associated misconceptions of middle-grade students, Bush and Karp (2013) assert that teachers must know how to “use multiple representations; identify, understand, and use tasks which are conceptually related; and have the ability to pose problems” (p. 627). All the representations above were included in this study. In order to build the conceptual understanding of students with MLD, teachers need to model (verbalize and demonstrate) the use of multiple representations of mathematical concepts.

Policy and Economic Implications

Although high-stakes testing and accountability are critical to the current mathematics education policies and agendas (Katsiyannis, Zhang, Ryan, & Jones, 2007; Powell, 2011; Stevens, Schulte, Elliott, Nese, & Tindal, 2015), interventions that enhance the mathematics performance of students with learning disabilities have received little attention from the research community, policy makers, and school leaders as compared to the field of reading (Gersten, Clarke, & Mazzocco, 2007). High-stakes standardized tests like the National Assessment of Educational Progress (NAEP) and the SAT heavily emphasize mathematics word problems. Students with learning disabilities continue to struggle with word problems (a principal component of high-stakes testing, Common Core State Standards for Mathematics, and one of the five Process Standards of NCTM). This may be in part because they have not received appropriate and sufficient mathematics intervention to develop their conceptual understanding and procedural fluency skills.

Advancing knowledge of instructional practices and evidence-based interventions for students with MLD requires adequate funding and attention from federal and state lawmakers. Full funding of the Elementary and Secondary Education Act (ESEA) and Individuals with Disabilities Education Act (IDEA) is needed for states and school districts to implement policy initiatives that foster equitable outcomes for all students, especially those at risk of academic failure. Without adequate funding dedicated to evidence-based practices and interventions, schools might be unduly pressured to limit the support they provide to students with MLD.

Beyond school walls, there are adverse economic consequences of poorly developed mathematical competencies (Geary, 2012; Hudson & Miller, 2006). For example, Geary (2012) found that for both men and women, “poor mathematics skills were associated with lower rates of full-time employment, higher rates of employment in low-paying manual occupations, more frequent periods of unemployment, and a lower ability to take advantage of employer offered training and thus lower rates of promotion.” (p. 2). In order to avert the future economic crisis associated with poor mathematics competencies, appropriate interventions should be provided to students who are experiencing difficulties in learning mathematics in K-12 settings. The present study contributes to our understanding of effective instructional practices that should be part of the intervention effort.

Limitation and Future Directions

There were five limitations in the present study that should be considered in future research. First, the study did not investigate the effect of verbalizations on long-term maintenance of students' conceptual understanding and procedural fluency related to solving word problems. The study results were collected over a period of two-three months for all students. Second, because all the students are reading at least two years below their current grade level, the teachers read aloud the word problems. Prior studies have identified reading ability as one of the major factors influencing mathematical problem solving (Fuchs & Fuchs, 2002; Fuchs et al., 2008; Swanson, 2006). Swanson (2006) implicated reading comprehension as a reliable predictor of both procedural and conceptual understanding. For the present study, it may be that the read-aloud accommodation might have influenced the results, contributing to the effect sizes. On the other hand, it may be that the results suggest that read aloud accommodation are an important necessary component of intervention in some cases. Third, the intervention was not implemented during regular classroom instruction. Future studies should examine the impact of student verbalizations within the context of regular classroom instruction. Fourth, the students solved one problem during the independent work time in each session. Future research should consider including more word problems.

Finally, it is still unclear why the verbalization of mathematical thinking resulted in more gains in conceptual understanding than procedural fluency. Investigating this outcome could be difficult because there have been mixed results regarding the shared or distinct aspects of cognitive processes responsible for these two strands of mathematical proficiency (Fuchs, Fuchs, Stuebing, Fletcher, Hamlett, & Lambert, 2008). The procedural fluency is mainly influenced by inhibitory control (a form of attention), visual-spatial working memory, and processing speed (Fuchs et al., 2008; Swanson, 2006). This assertion sounds logical because procedural fluency is associated with the “knowledge of the rules and procedures used in carrying out mathematical processes and also the symbolism used to represent mathematics.” (Van de Walle, Karp, and Bay-Williams, 2010, p. 24). On the other hand, Swanson (2006) submits that language ability, reading skill, and concept formation may influence student’s conceptual understanding. Further research is needed to better understand how verbalization (an indicator of metacognitive regulation and processes) affects the intertwined domains of procedural fluency and conceptual understanding in the problem-solving process.

Despite the fifth limitation, the present study validates the effectiveness of verbalizations on the mathematical problem-solving abilities of elementary students with MLD. Van de Walle, Karp, and Bay-Williams (2010) caution that “the common practice of teaching procedures in the absence of conceptual understanding leads to errors and dislikes of mathematics” (p. 24). In this regard, the present study posits that teachers can foster both conceptual and procedural understanding by providing the students with opportunities to verbalize their thinking while solving word problems.

References

Anderson, N., Chapin, S., & O'Connor, C. (2011). Classroom discussions in math: A facilitator's guide to support professional learning of discourse and the Common Core, Grades K–6.

Alarcon, M., DeFries, J. C., Light, J. G., & Pennington, B. F. (1997). A twin study of mathematics disability. *Journal of Learning Disabilities,* 30(6), 617-623. DOI: 10.1177/002221949703000605

Aikens, N. L., & Barbarin, O. (2008). Socioeconomic differences in reading trajectories: The contribution of family, neighborhood, and school contexts. *Journal of Educational Psychology,* 100(2), 235.

American Psychiatric Association. *Diagnostic and statistical manual of mental disorders*. 4. Washington, DC: Author; 2004.

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews Neuroscience*, 9, 278–291. DOI: 10.1038/nrn2334

Appleton, J. J., Christenson, S. L., Kim, D., & Reschly, A. L. (2006). Measuring cognitive and psychological engagement: Validation of the student engagement instrument. *Journal of School Psychology,* 44(5), 427-445. DOI: 10.1016/j.jsp.2006.04.002

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics,* 52(3), 215-241. DOI: 10.1023/A:1024312321077

Atkinson, R. C., & Shiffrin, R. M. (1968). Human memory: A proposed system and its control processes. *Psychology of learning and motivation*, 2, 89-195. DOI: 10.1016/S0079-7421(08)60422-3

Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal*, 103, 51-73. DOI: 10.1086/499715

Berch, D. B., & Mazzocco, M. M. (2007). *Why Is Math So Hard for Some Children? The Nature and Origins of Mathematical Learning Difficulties and Disabilities*. Brookes Publishing Company. PO Box 10624, Baltimore, MD 21285.

Borman, G. D., & Overman, L. T. (2004). Academic resilience in mathematics among poor and minority students. *The Elementary School Journal*, *104*(3), 177-195. DOI: 10.1086/499748

Borckardt, J. J., Nash, M. R., Murphy, M. D., Moore, M., Shaw, D., & O'Neil, P. (2008). Clinical practice as natural laboratory for psychotherapy research: A guide to case-based time-series analysis. *American Psychologist, 63*(2), 77. DOI: 10.1037/0003-066X.63.2.77

Booth, J. L., Barbieri, C., Eyer, F., & Paré-Blagoev, E. J. (2014). Persistent and Pernicious Errors in Algebraic Problem Solving. *Journal of Problem Solving*, *7*(1), 3. DOI: DOI: 10.7771/1932-6246.1161

Booth, R. D., & Thomas, M. O. (1999). Visualization in mathematics learning: Arithmetic problem-solving and student difficulties. *The Journal of Mathematical Behavior*, 18(2), 169-190. DOI: 10.1016/S0732-3123(99)00027-9

Bosson, M., Hessels, M., Hessels-Schlatter, C., Berger, J-L., Kipfer, N., & Büchel, F. (2010). Strategy acquisition by children with general learning difficulties through metacognitive training. *Australian Journal of Learning Difficulties,* 15 (1), 13-34. DOI: 10.1080/19404150903524523

Bottge, B. A., Rueda, E., LaRoque, P. T., Serlin, R. C., & Kwon, J. (2007). Integrating Reform-Oriented Math Instruction in Special Education Settings. *Learning Disabilities Research & Practice*, *22*(2), 96-109. doi:10.1111/j.1540-5826.2007.00234.x

Bradley, R. H., & Corwyn, R. F. (2002). Socioeconomic status and child development. *Annual Review of Psychology,* 53(1), 371-399. DOI: 10.1146/annurev.psych.53.100901.135233

Bråten, I., & Throndsen, I. S. (1998). Cognitive Strategies in Mathematics, Part II: teaching a more advanced addition strategy to an eight-year-old girl with learning difficulties. *Scandinavian Journal of Educational Research*, *42*(2), 151-175. DOI: 10.1080/0031383980420204

Bray, M. A., & Kehle, T. J. (Eds.). (2013). *The Oxford handbook of school psychology*. Oxford University Press.

Brossart, D. F., Parker, R. I., Olson, E. A., & Mahadevan, L. (2006). The relationship between visual analysis and five statistical analyses in a simple AB single-case research design. *Behavior Modification, 30*(5), 531-563. DOI: 10.1177/0145445503261167

Browder, D. M., Spooner, F., Ahlgrim-Delzell, L., Harris, A. A., & Wakemanxya, S. (2008). A meta-analysis on teaching mathematics to students with significant cognitive disabilities. *Exceptional Children, 74*(4), 407-432. DOI: 10.1177/001440290807400401

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology,* 19(3), 273-293. DOI: 10.1207/S15326942DN1903_3

Bush, S. B., & Karp, K. S. (2013). Prerequisite algebra skills and associated misconceptions of middle grade students: A review. *The Journal of Mathematical Behavior*, *32*(3), 613-632. DOI: 10.1016/j.jmathb.2013.07.002

Butterworth, B. (2005). The development of arithmetical abilities. *Journal of Child Psychology and Psychiatry, 46*(1), 3-18. DOI: 10.1111/j.1469-7610.2004.00374.x

Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in cognitive sciences*, *14*(12), 534-541. DOI: 10.1016/j.tics.2010.09.007

Byiers, B. J., Reichle, J., & Symons, F. J. (2012). Single-subject experimental design for evidence-based practice. *American Journal of Speech-Language Pathology, 21*(4), 397-414. DOI: 10.1044/1058-0360(2012/11-0036.

Byrnes, J. P. (2003). Factors predictive of mathematics achievement in White, Black, and Hispanic 12th graders. *Journal of Educational Psychology, 95*(2), 316-326. DOI: 10.1037/0022-0663.95.2.316

Calhoon, M. B., & Fuchs, L. S. (2003). The effects of peer-assisted learning strategies and curriculum-based measurement on the mathematics performance of secondary students with disabilities. *Remedial and Special Education, 24*(4), 235-245. DOI: 10.1177/07419325030240040601

Campbell, J. M. (2004). Statistical comparison of four effect sizes for single-subject designs. *Behavior Modification, 28*(2), 234-246. DOI: 10.1177/0145445503259264

Carr, J., & Bertrando, S. (2012). Top 10 Instructional Strategies for Struggling Students. *Leadership*, *42*(1), 24.

Case, L., Harris, K., & Graham, S. (1992). Improving the mathematical problem-solving skills of students with learning disabilities: Self-regulated strategy development. *The Journal of Special Education,* 26(1), 1–19. DOI: 10.1177/002246699202600101

Chatterji, M. (2006). Reading achievement gaps, correlates, and moderators of early reading achievement: Evidence from the Early Childhood Longitudinal Study (ECLS) kindergarten to first grade sample. *Journal of Educational Psychology,* 98(3), 489-507. DOI: 10.1037/0022-0663.98.3.489

Clarke, B., Smolkowski, K., Baker, S. K., Fien, H., Doabler, C. T., & Chard, D. J. (2011). The impact of a comprehensive Tier I core kindergarten program on the achievement of students at risk in mathematics. *The Elementary School Journal*, *111*(4), 561-584. DOI: 10.1177/0022219414538514

Clark, R. E., & Salomon, G. (1986). Media in teaching. *Handbook of research on teaching*, *3*, 464-478.

Codding, R. S., Burns, M. K., & Lukito, G. (2011). Meta-analysis of mathematic basic-fact fluency interventions: A component analysis. *Learning Disabilities Research & Practice*, *26*(1), 36-47. DOI: 10.12691/education-4-4-1

Cohen, J. (1988). Statistical power analysis for the behavioral sciences. Lawrence Earlbaum Associates. *Hillsdale, NJ*, 20-26.

Cole, J., & Wasburn-Moses, L. (2010). Going beyond the math wars: A special educator's guide to understanding and assisting with inquiry-based teaching in mathematics. *Teaching Exceptional Children,* 42(4), 14-20. DOI: 10.1177/0731948711429726

Coleman, J. S., Campbell, E. Q., Hobson, C. J., McPartland, J., Mood, A. M., Weinfeld, F. D., & York, R. L. (1966). Equality of Educational Opportunity (Washington, DC, Government Printing Office). *Journal of the Royal Statistical Society, Series A*, *156*, 938.

Cortiella, C., & Horowitz, S. H. (2014). The state of learning disabilities: Facts, trends and emerging issues. *New York: National Center for Learning Disabilities*.

Craik, F. I., & Lockhart, R. S. (1972). Levels of processing: A framework for memory research. *Journal of verbal learning and verbal behavior,* 11(6), 671-684. DOI: 10.1016/S0022-5371(72)80001-X

Dahl, G. B., & Lochner, L. (2005). The impact of family income on child achievement (Working paper 11279). *Cambridge, MA: National Bureau of Economic Research.*

Davis, W., & Hajicek, J. (1985). Effects of self-instructional training and strategy training on a mathematics task with severely behaviorally disordered students. *Behavioral Disorders*, 10(3), 211–218. DOI: 10.1177/019874298501000403

Del'Homme, M., Kim, T. S., Loo, S. K., Yang, M. H., & Smalley, S. L. (2007). Familial association and frequency of learning disabilities in ADHD sibling pair families. *Journal of Abnormal Child Psychology*, 35(1), 55-62. DOI: 10.1007/s10802-006-9080-5

Deshler, D. D., Schumaker, J. B., Lenz, B. K., Bulgren, J. A., Hock, M. F., Knight, J., & Ehren, B. J. (2001). Ensuring content-area learning by secondary students with learning disabilities. *Learning Disabilities Research & Practice,* 16(2), 96-108. DOI: 10.1111/0938-8982.00011

Desoete, A., Roeyers, H., & Buysse, A. (2001). Metacognition and mathematical problem solving in grade 3. *Journal of Learning Disabilities,* 34(5), 435-449. DOI: 10.1177/002221940103400505

Driscoll, M. (1999). *Fostering Algebraic Thinking: A Guide for Teachers, Grades 6-10*. Heinemann, 361 Hanover Street, Portsmouth, NH 03801-3912.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Brooks-Gunn, J. (2007). School readiness and later achievement. *Developmental Psychology,* 43(6), 1428. DOI: 10.1037/0012-1649.43.6.1428

Dunn, C., Chambers, D., & Rabren, K. (2004). Variables affecting students' decisions to drop out of school. *Remedial and Special Education*, *25*(5), 314-323. DOI: 10.1177/07419325040250050501

Durlak, J. A. (2009). How to select, calculate, and interpret effect sizes. *Journal of pediatric psychology*, *34*(9), 917-928. DOI: 10.1093/jpepsy/jsp004

Durlak, J. A., & DuPre, E. P. (2008). Implementation matters: A review of research on the influence of implementation on program outcomes and the factors affecting implementation. *American Journal of Community Psychology, 41*, 327–350. DOI: 10.1007/s10464-008-9165-0

Dusenbury, L., Brannigan, R., Falco, M., & Hansen, W. B. (2003). A review of research on fidelity of implementation: Implications for drug abuse prevention in school settings. *Health Education Research,* 18, 237–256. DOI: 10.1093/her/18.2.237

Eckes, S. E., & Swando, J. (2009). Special education subgroups under NCLB: Issues to consider. *Teachers College Record*, *111*(11), 2479.

Evans, G. W., & English, K. (2002). The environment of poverty: Multiple stressor exposure, psychophysiological stress, and socioemotional adjustment. *Child Development*, 73(4), 1238-1248. DOI: 10.1111/1467-8624.00469

Evans, G. W. (2004). The environment of childhood poverty. *American Psychologist,* 59(2), 77. DOI: 10.1037/0003-066X.59.2.77

Fischer, U., Moeller, K., Cress, U., & Nuerk, H. C. (2013). Interventions supporting children's mathematics school success: A meta-analytic review. DOI: 10.1027/1016-9040/a000141

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive–developmental inquiry. *American Psychologist,* 34(10), 906. DOI: 10.1037/0003-066X.34.10.906

Fletcher, J., Lyon, G., Fuchs, L., & Barnes, MA (2007). *Learning Disabilities: From Identification to Intervention*. New York: Guilford Press.

Friend, M., Cook, L., Hurley-Chamberlain, D., & Shamberger, C. (2010). Co-teaching: An illustration of the complexity of collaboration in special education. *Journal of Educational and Psychological Consultation, 20*(1), 9-27. DOI: 10.1177/0888406412474996

Fuchs, L. S., Fuchs, D., Powell, S. R., Seethaler, P. M., Cirino, P. T., & Fletcher, J. M. (2008). Intensive intervention for students with mathematics disabilities: Seven principles of effective practice. *Learning Disability Quarterly*, *31*(2), 79-92. DOI: 10.2307/20528819

Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition? *Journal of educational psychology*, *100*(1), 30. DOI: 10.1037/0022-0663.100.1.30

Fuchs, L. S., Fuchs, D., & Prentice, K. (2004). Responsiveness to mathematical problem-solving instruction: Comparing students at risk of mathematics disability with and without risk of reading disability. *Journal of Learning Disabilities, 37,* 293–306. DOI: 10.1177/00222194040370040201

Fuchs, L. S., & Fuchs, D. (2002). Mathematical problem-solving profiles of students with mathematics disabilities with and without comorbid reading disabilities. *Journal of learning disabilities*, *35*(6), 564-574. DOI: 10.1177/00222194020350060701

Garrity, M. L., Luiselli, J. K., & McCollum, S. A. (2008). Effects of a supervisory intervention on assessment of inter-observer agreement by educational service providers. *Behavioral Interventions, 23*(2), 105-112. DOI: 10.1002/bin.258

Gay, G. (2002). Culturally responsive teaching in special education for ethnically diverse students: Setting the stage. *International Journal of Qualitative Studies in Education, 15*(6), 613-629. DOI: 10.1080/0951839022000014349

Geary, D. C. (2010). Mathematical disabilities: Reflections on cognitive, neuropsychological, and genetic components. *Learning and individual differences*, *20*(2), 130-133. DOI: 10.1016/j.lindif.2009.10.008

Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology, 74*(3), 213-239. DOI: 10.1006/jecp.1999.2515

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, 37(1), 4-15. DOI: 10.1006/jecp.1999.2515

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78*(4), 1343-1359. DOI: 10.1111/j.1467-8624.2007.01069.x

Geary, D. C. (2011). Consequences, characteristics, and causes of mathematical learning disabilities and persistent low achievement in mathematics. *Journal of Developmental and Behavioral Pediatrics, 32*(3), 250-263. DOI: 10.1097/DBP.0b013e318209edef

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79*, 1202–1242. DOI: 10.3102/0034654309334431

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2008). Mathematics instruction for students with learning disabilities or difficult learning mathematics: A synthesis of the intervention research. Portsmouth, NH: RMC Research Corporation, Center on Instruction. DOI: 10.3102/0034654309334431

Gersten, R., Clarke, B., & Mazzocco, M. (2007). Historical and contemporary perspectives on mathematical learning disabilities. In D. B. Berch & M. M. M. Mazzocco (Eds.), Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities (pp. 7–29). Baltimore: Brooks.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38*(4), 293-304. DOI: 10.1177/00222194050380040301

Gersten, R., Fuchs, L. S., Compton, D., Coyne, M., Greenwood, C., & Innocenti, M. S. (2005). Quality indicators for group experimental and quasi-experimental research in special education. *Exceptional Children, 71*(2), 149–164. DOI: 10.1177/1053451208321452

Greene, G. (1999). Mnemonic multiplication fact instruction for students with learning disabilities. *Learning Disabilities Research & Practice, 14*(3), 141-148. DOI: 10.1207/sldrp1403_2

Griner, A. C., & Stewart, M. L. (2013). Addressing the achievement gap and disproportionality through the use of culturally responsive teaching practices. *Urban Education*, *48*(4), 585-621. DOI: 10.1177 /0042085912456847

Gross, J., Hudson, C., & Price, D. (2009). The long-term costs of numeracy difficulties. Every Child a Chance Trust and KPMG, London.

Guo, G., & Harris, K. M. (2000). The mechanisms mediating the effects of poverty on children's intellectual development. *Demography, 37*(4), 431-447. DOI: 10.1353/dem.2000.0005

Hackman, D. A., Farah, M. J., & Meaney, M. J. (2010). Socioeconomic status and the brain: Mechanistic insights from human and animal research. *Nature Reviews Neuroscience*, *11*(9), 651-659. DOI:10.1038/nrn2897

Hagermoser Sanetti, L. M., Fallon, L. M., & Collier-Meek, M. A. (2013). Increasing teacher treatment integrity through performance feedback provided by school personnel. *Psychology in the Schools*, *50*(2), 134-150. DOI: 10.1016/j.jsp.2015.03.002

Sanetti, L. M. H., & Kratochwill, T. R. (2009). Toward developing a science of treatment integrity: Introduction to the special series. *School Psychology Review*, *38*(4), 445.

Harbour, K. E., Karp, K. S., & Lingo, A. S. (2016). Inquiry to Action: Diagnosing and Addressing Students' Relational Thinking About the Equal Sign. *TEACHING Exceptional Children*, *49*(2), 126-133. DOI: 10.1177/0040059916673310

Harlacher, J. E., Sanford, A., & Nelson Walker, N. (2014). Distinguishing between tier 2 and tier 3 instruction in order to support implementation of RTI. *RTI Action Network Monthly Newsletter*.

Hasselbring, T. S. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children*, *20*(6), 1-7.

Hedges, L. V., Pustejovsky, J. E., & Shadish, W. R. (2013). A standardized mean difference effect size for multiple baseline designs across individuals. *Research Synthesis Methods*, *4*(4), 324-341. DOI: 10.1002/jrsm.1086

Hedges, L. V., & Hedberg, E. C. (2007). Intraclass correlation values for planning group-randomized trials in education. *Educational Evaluation and Policy Analysis*, *29*(1), 60-87. DOI: 10.3102/0162373707299706

Hook, C. L., & DuPaul, G. J. (1999). Parent tutoring for students with Attention-Deficit/Hyperactivity Disorder: Effects on reading performance at home and school. *School Psychology Review, 28*(1), 60-75.

Horner, R. H., Carr, E. G., Halle, J., McGee, G., Odom, S., & Wolery, M. (2005). The use of single-subject research to identify evidence-based practice in special education. *Exceptional children*, *71*(2), 165-179. DOI: 10.1177/001440290507100203

Howard, T. C. (2003). Culturally relevant pedagogy: Ingredients for critical teacher reflection. *Theory into Practice,* 42(3), 195-202. DOI: 10.1207/s15430421tip4203_5

Hutchinson, N. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disability Quarterly, 16*(1), 34-63. DOI: 10.2307/1511158

Individuals With Disabilities Education Act, 20 U.S.C. § 1400 (2004).

Jayanthi, M., Gersten, R., & Baker, S. (2008). Mathematics Instruction for Students with Learning Disabilities or Difficulty Learning Mathematics: A Guide for Teachers. *Center on Instruction*.

Jitendra, A., DiPipi, C. M., & Perron-Jones, N. (2002). An exploratory study of schema-based word-problem-solving instruction for middle school students with learning disabilities: An emphasis on conceptual and procedural understanding. *The Journal of Special Education,* 36, 23–38. DOI: 10.1177/00224669020360010301

Johnson, E. S., Humphrey, M., Mellard, D. F., Woods, K., & Swanson, H. L. (2010). Cognitive processing deficits and students with specific learning disabilities: A selective meta-analysis of the literature. *Learning Disability Quarterly*, *33*(1), 3-18. 10.1177/073194871003300101

Jordon, N. C., Kaplan, D., & Hanich, L. B. (2002). Achievement growth in children with learning difficulties in mathematics: Findings of a two-year longitudinal study. *Journal of educational psychology*, *94*(3), 586. DOI: 10.1037/0022-0663.94.3.586

Karagiannakis, G., Baccaglini-Frank, A., & Papadatos, Y. (2014). Mathematical learning difficulties subtypes classification. *Frontiers in Human Neuroscience,* 8, 57. DOI: 10.3389/fnhum.2014.00057

Karagiannakis, G., & Baccaglini-Frank, A. (2014). The DeDiMa battery: A tool for identifying students' mathematical learning profiles. *Health Psychology Review, 2*(4). DOI: 10.5114/hpr.2014.46329

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2015). 12 Math Rules That Expire in the Middle Grades. *Mathematics Teaching in the Middle School*, *21*(4), 208-215. DOI: 10.5951/mathteacmiddscho.21.4.0208

Karp, K. S., & Voltz, D. L. (2000). Weaving mathematical instructional strategies into inclusive settings. *Intervention in School and Clinic*, *35*(4), 206-215. DOI: 10.1177/105345120003500402

Katsiyannis, A., Zhang, D., Ryan, J. B., & Jones, J. (2007). High-stakes testing and students with disabilities: Challenges and promises. *Journal of Disability Policy Studies*, *18*(3), 160-167. DOI: 10.1177/10442073070180030701

Ketterlin-Geller, L. R., Chard, D. J., & Fien, H. (2008). Making connections in mathematics conceptual mathematics intervention for low-performing students. *Remedial and Special Education,* 29(1), 33-45. DOI: 10.1177/0741932507309711

Kingsdorf, S., & Krawec, J. (2014). Error analysis of mathematical word problem solving across students with and without learning disabilities. *Learning Disabilities Research & Practice*, *29*(2), 66-74. DOI: 10.1111/ldrp.12029

Kosc, L. (1970). Psychology and psychopathology of mathematical abilities. *Studia psychologica*, *12*(2), 159-162. DOI: 10.1177/002221947400700309

Kratochwill, T. R., Hitchcock, J., Horner, R. H., Levin, J. R., Odom, S. L., Rindskopf, D. M., & Shadish, W. R. (2010). Single-case designs technical documentation. *What works clearinghouse*. DOI: 10.1177/0741932512452794

Kratochwill, T. R., Hitchcock, J. H., Horner, R. H., Levin, J. R., Odom, S. L., Rindskopf, D. M., & Shadish, W. R. (2013). Single-case intervention research design standards. *Remedial and Special Education,* 34, 26–38. DOI: 10.1177/0741932512452794

Kroesbergen, E. H., & Van Luit, J. E. (2003). Mathematics interventions for children with special educational needs a meta-analysis. *Remedial and Special Education,* 24(2), 97-114. DOI: 10.1177/07419325030240020501

Lee, J. (2012). Educational equity and adequacy for disadvantaged minority students: School and teacher resource gaps toward national mathematics proficiency standard. *The Journal of Educational Research,* 105(1), 64-75. DOI: 10.1080/00220671.2010.519409

Lee, J. (2002). Racial and ethnic achievement gap trends: Reversing the progress toward equity? *Educational researcher*, *31*(1), 3-12. DOI: 10.3102/0013189X031001003

Leon, J., & Pepe, H. (1983). Self-instructional training: Cognitive behavior modification for remediating arithmetic deficits. *Exceptional Children,* 55–60. DOI: 10.1177/001440298305000107

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. *Problems of representation in the teaching and learning of mathematics*, *21*, 33-40.

Leviton, L. C., & Lipsey, M. W. (2007). A big chapter about small theories: Theory as method: Small theories of treatments. *New Directions for Evaluation,* 2007(114), 27–62. DOI: DOI: 10.1002/ev.224

Maccini, P., Mulcahy, C. A., & Wilson, M. G. (2007). A follow-up of mathematics interventions for secondary students with learning disabilities. *Learning Disabilities Research & Practice*, *22*(1), 58-74. DOI: 10.1111/j.1540-5826.2007.00231.x

Maheady, L., Harper, G. F., & Mallette, B. (2001). Peer-mediated instruction and interventions and students with mild disabilities. *Remedial and Special Education,* 22(1), 4-14. DOI: 10.1177/074193250102200102

Manalo, E., Bunnell, J. K., & Stillman, J. A. (2000). The use of process mnemonics in teaching students with mathematics learning disabilities. *Learning Disability Quarterly,* 23(2), 137-156. DOI: 10.2307/1511142

Marita, S., & Hord, C. (2017). Review of Mathematics Interventions for Secondary Students With Learning Disabilities. *Learning Disability Quarterly*, *40*(1), 29-40. DOI: 10.1177/0731948716657495

Mazzocco, M. M. (2001). Math learning disability and math LD subtypes: Evidence from studies of turner syndrome, fragile X syndrome, and neurofibromatosis type 1. *Journal of Learning Disabilities,* 34(6), 520-533. DOI: 10.1177/002221940103400605

Mazzocco, M. M., & Myers, G. F. (2003). Complexities in identifying and defining mathematics learning disability in the primary school-age years. *Annals of Dyslexia,* 53(1), 218-253. DOI: 10.1007/s11881-003-0011-7

Mazzocco, M. M. (2005). Challenges in identifying target skills for math disability screening and intervention. *Journal of Learning Disabilities,* 38(4), 318-323. DOI: 10.1177/00222194050380040701

Mazzocco, M. M. M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. B. Berch & M. M. M. Mazzocco (Eds.), Why is math so hard for some children? (pp 29–47). Baltimore, MD: Brooks.

McClelland, J. L. (1988). Connectionist models and psychological evidence. *Journal of Memory and Language, 27*(2), 107-123. DOI: 10.1016/0749-596X(88)90069-1

McDougal, J. L., Graney, S. B., Wright, J. A., & Ardoin, S. P. (2009). *RTI in practice: A practical guide to implementing effective evidence-based interventions in your school*. John Wiley & Sons.

McLean, J. F., & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology,* 74(3), 240-260. DOI: 10.1006/jecp.1999.2516

Mellard, D., McKnight, M., & Jordan, J. (2010). RTI tier structures and instructional intensity. *Learning Disabilities Research & Practice*, *25*(4), 217-225. DOI: 10.1111/j.1540-5826.2010.00319.x

Methe, S. A., Kilgus, S. P., Neiman, C., & Riley-Tillman, T. C. (2012). Meta-analysis of interventions for basic mathematics computation in single-case research. *Journal of Behavioral Education*, *21*(3), 230-253. DOI: 10.1007/s10864-012-9161-1

Milsom, A., & Glanville J. L. (2010). Factors mediating the relationship between social skills and academic grades in a sample of students diagnosed with learning disabilities or emotional disturbance. *Remedial and Special Education,* 31(4), 241-251. DOI: 10.1177/0741932508327460

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological review,* 63(2), 81. DOI: dx.doi.org/10.1037/h0043158

Mohd Syah, N. E., Hamzaid, N. A., Murphy, B. P., & Lim, E. (2016). Development of computer play pedagogy intervention for children with low conceptual understanding in basic mathematics operation using the dyscalculia feature approach. *Interactive Learning Environments*, *24*(7), 1477-1496. 10.1080/10494820.2015.1023205

Montague, M. (2008). Self-regulation strategies to improve mathematical problem solving for students with learning disabilities. *Learning Disability Quarterly, 31*(1), 37-44. DOI: 10.2307/30035524

Montague, M. (2003). *Solve it! A practical approach to teaching mathematical problem-solving skills.* Reston, VA: Exceptional Innovations.

Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities,* 25(4), 230-248. DOI: 10.1177/002221949202500404

Morris, C. D., Bransford, J. D., & Franks, J. J. (1977). Levels of processing versus transfer appropriate processing. *Journal of verbal learning and verbal behavior,* 16(5), 519-533. DOI: 10.1080/10862968509547541

Murphy, M. M., Mazzocco, M. M., Hanich, L. B., & Early, M. C. (2007). Cognitive characteristics of children with mathematics learning disability (MLD) vary as a function of the cutoff criterion used to define MLD. *Journal of Learning Disabilities,* 40(5), 458-478. DOI: 10.1177/00222194070400050901

Nagro, S. A., & Cornelius, K. E. (2013). Evaluating the evidence base of video analysis: A special education teacher development tool. *Teacher Education and Special Education*, *36*(4), 312-329. DOI: 10.1177/0888406413501090

Naglieri, J., & Johnson, D. (2000). Effectiveness of a cognitive strategy intervention in improving arithmetic computation based on the PASS theory. *Journal of Learning Disabilities,* 33(6), 591-597. DOI: 10.1177/002221940003300607

National Mathematics Advisory Panel. (2008). Foundations for Success: The Final Report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education.

National Assessment of Educational Progress (2015). Mathematics Assessment. Washington, DC: U. S. Department of Education, Institute of Education Sciences, National Center for Education Statistics.

National Assessment of Educational Progress (2013). Mathematics Assessment. Washington, DC: U. S. Department of Education, Institute of Education Sciences, National Center for Education Statistics.

Nelson, P. M., Burns, M. K., Kanive, R., & Ysseldyke, J. E. (2013). Comparison of a math fact rehearsal and a mnemonic strategy approach for improving math fact fluency. *Journal of school psychology*, *51*(6), 659-667. DOI: 10.1016/j.jsp.2013.08.003

Nelson, M. C., Cordray, D. S., Hulleman, C. S., Darrow, C. L., & Sommer, E. C. (2012). A procedure for assessing intervention fidelity in experiments testing educational and behavioral interventions. *The Journal of Behavioral Health Services & Research,* 39, 374–396. DOI: 10.1007/s11414-012-9295-x

Noell, G. H., Witt, J. C., Slider, N. J., & Connell, J. E. (2005). Treatment implementation following behavioral consultation in schools: A comparison of three follow-up strategies. *School Psychology Review, 34*(1), 87. DOI: 10.1037/h0100816

Oakes, J. M., & Rossi, P. H. (2003). The measurement of SES in health research: Current practice and steps toward a new approach. *Social Science & Medicine*, 56(4), 769-784. DOI: 10.1016/S0277-9536(02)00073-4

Olive, M. L., & Smith, B. W. (2005). Effect size calculations and single subject designs. *Educational Psychology, 25*(2-3), 313-324. DOI: 10.1080/0144341042000301238

Owen, R. L., & Fuchs, L. S. (2002). Mathematical problem-solving strategy instruction for third-grade students with learning disabilities. *Remedial and Special Education,* 23(5), 268-278. DOI: 10.1177/07419325020230050201

O'Donnell, C. (2008). Defining, conceptualizing, and measuring fidelity of implementation and its relationship to outcomes in K-12 curriculum intervention research. *Review of Educational Research,* 78, 33-84. DOI: 10.3102/0034654307313793

Parker, R. I., Cryer, J., & Byrns, G. (2006). Controlling baseline trend in single-case research. *School Psychology Quarterly, 21*(4), 418. DOI: 10.1037/h0084131

Parmar, R. (1992). Protocol analysis of strategies used by students with mild disabilities when solving arithmetic word problems. *Diagnostique*, *17*(4), 227-243. DOI: 10.1177/153450849201700401

Passolunghi, M. C., & Siegel, L. S. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology,* 88(4), 348-367. DOI: 10.1016/j.jecp.2004.04.002

Pellegrino, J. W., & Goldman, S. R. (1987). Information processing and elementary mathematics. *Journal of Learning Disabilities,* 20, 23–32. DOI: 10.1177/002221948702000302

Powell, S. R. (2011). Solving word problems using schemas: A review of the literature. *Learning Disabilities Research & Practice*, *26*(2), 94-108. DOI: 10.1111/j.1540-5826.2011.00329.x

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and individual differences*, *20*(2), 110-122. DOI: 10.1016/j.lindif.2009.10.005

Reschly, A. L., & Christenson, S. L. (2006). Prediction of dropout among students with mild disabilities A case for the inclusion of student engagement variables. *Remedial and Special Education*, 27(5), 276-292. DOI: 10.1177/07419325060270050301

Riccomini, P. J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly*, *28*(3), 233-242. DOI: 10.2307/1593661

Rittle-Johnson, B., & Schneider, M. (2014). Developing conceptual and procedural knowledge of mathematics. *Oxford handbook of numerical cognition*, 1102-1118.

Rosenzweig, C., Krawec, J., & Montague, M. (2011). Metacognitive strategy use of eighth-grade students with and without learning disabilities during mathematical problem solving: A think-aloud analysis. *Journal of Learning Disabilities, 44(6),* 508-520. DOI: 10.1177/0022219410378445

Ross, S. G., & Begeny, J. C. (2014). Single-case effect size calculation: Comparing regression and non-parametric approaches across previously published reading intervention data sets. *Journal of School Psychology, 52*(4), 419-431. DOI: 10.1016/j.jsp.2014.06.003

Saldaña, J. (2009). An introduction to codes and coding. *The coding manual for qualitative researchers*. Thousand Oaks, CA: Sage.

Salminen, J., Koponen, T., Räsänen, P., & Aro, M. (2015). Preventive support for kindergarteners most at-risk for mathematics difficulties: Computer-assisted intervention. *Mathematical Thinking and Learning*, *17*(4), 273-295. DOI:10.1080/10986065.2015.1083837

Saunders, R. P., Evans, M. H., & Joshi, P. (2005). Developing a process-evaluation plan for assessing health promotion program implementation: *A how-to guide. Health Promotion Practice,* 6, 134–147. DOI: 10.1177/1524839904273387

Schulte, A. C., & Stevens, J. (2015). Once, sometimes, or always in special education: Mathematics growth and achievement gaps. *Exceptional Children,* 81(3), 370–387. DOI: 10.1177/0014402914563695

Schunk, D. H. (2012). Learning theories: An educational perspective (sixth. ed.). Upper Saddle River, NJ: Merrill/Prentice Hall.

Schunk, D., & Cox, P. (1986). Strategy training and attributional feedback with learning disabled students. *Journal of Educational Psychology*, 78(3), 201-209. DOI: 10.1037/0022-0663.78.3.201

Shalev, R. S., Manor, O., & Gross-Tsur, V. (2005). Developmental dyscalculia: A prospective six-year follow-up. *Developmental Medicine & Child Neurology,* 47(02), 121-125. DOI: 10.1017/S0012162205000216

Shalev, R. S. (2004). Developmental dyscalculia. *Journal of child neurology*, *19*(10), 765-771. DOI: 10.1177/08830738040190100601

Shuell, T. J. (1986). Cognitive conceptions of learning. *Review of Educational Research,* 56, 411–436. DOI: 10.3102/00346543056004411

Shadish, W., Cook, T., & Campbell, D. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Boston, MA: Houghton Mifflin.

Shadish, W. R., Rindskopf, D. M., & Hedges, L. V. (2008). The state of the science in the meta-analysis of single-case experimental designs. *Evidence-Based Communication Assessment and Intervention, 2*(3), 188-196. DOI: 10.1080/17489530802581603

Skinner, B. F. (1953). *Science and human behavior*. New York: Free Press.

Smith, J. D. (2012). Single-case experimental designs: A systematic review of published research and current standards. *Psychological Methods, 17*(4), 510. DOI: 10.1037/a0029312

Sirin, S. R. (2005). Socioeconomic status and academic achievement: A meta-analytic review of research. *Review of Educational Research,* 75(3), 417-453. DOI: 10.12691/education-4-6-6

Stevens, J. J., Schulte, A. C., Elliott, S. N., Nese, J. F., & Tindal, G. (2015). Growth and gaps in mathematics achievement of students with and without disabilities on a statewide achievement test. *Journal of school psychology*, *53*(1), 45-62. DOI: 10.1016/j.jsp.2014.11.001

Strickland, T. K., & Maccini, P. (2013). The effects of the concrete–representational–abstract integration strategy on the ability of students with learning disabilities to multiply linear expressions within area problems. *Remedial and Special Education*, *34*(3), 142-153. DOI: 10.1177/0741932512441712

Strosberg, M. A., & Wholey, J. S. (1983). Evaluability assessment: From theory to practice in the department of health and human services. *Public Administration Review,* 43, 66–71. DOI: 10.1177/1098214007304589

Swanson, H. L. (2006). Cross-sectional and incremental changes in working memory and mathematical problem solving. *Journal of Educational Psychology,* 98, 265–281. DOI: 10.1037/0022-0663.98.2.265

Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of experimental child psychology*, *79*(3), 294-321. DOI: 10.1006/jecp.2000.2587

Swanson, H. L. (1990). Influence of metacognitive knowledge and aptitude on problem solving. *Journal of educational psychology*, *82*(2), 306. DOI: 10.1037//0022-0663.82.2.306

Taiwo, E. (2015). *Addressing the underachievement of students with mathematical learning disabilities*. Unpublished Manuscript, Johns Hopkins University.

Thomas, K. R. (2006). Students THINK: A framework for improving problem solving. *Teaching Children Mathematics, 13*(2), 86.

Thorndike, E. L. (1913). *The psychology of learning* (Vol. 2). Teachers College, Columbia University.

Tournaki, N. (2003). The differential effects of teaching addition through strategy instruction versus drill and practice to students with and without learning disabilities. *Journal of Learning Disabilities,* 36(5), 449-458. DOI: 10.1177/00222194030360050601

Van de Walle, J. A., Karp, K. S., & Williams, J. M. B. (2013). *Elementary and middle school mathematics. Teaching development*. Boston: Pearson.

Veenman, M. V., Van Hout-Wolters, B. H., & Afflerbach, P. (2006). Metacognition and learning: Conceptual and methodological considerations. *Metacognition and Learning,* 1(1), 3-14. DOI: 10.1007/s11409-006-6893-0

von Hippel, P. T. (2009). Achievement, learning, and seasonal impact as measures of school effectiveness: It's better to be valid than reliable. *School Effectiveness and School Improvement,* 20(2), 187-213. DOI: 10.1080/09243450902883888

Wang, A. Y., Fuchs, L. S., & Fuchs, D. (2016). Cognitive and linguistic predictors of mathematical word problems with and without irrelevant information. *Learning and individual differences*, *52*, 79-87. DOI: 10.1016/j.lindif.2016.10.015

Watson, S. M., & Gable, R. A. (2013). Unraveling the complex nature of mathematics learning disability implications for research and practice. *Learning Disability Quarterly, 36*(3), 178-187. DOI: 10.1177/0731948712461489

Witzel, B.S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research and Practice,* 18, 121–131. DOI: 10.1177/0022219414564220

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly*, 29, 269–289. DOI: 10.2307/30035554

Xin, Y. P., Jitendra, A. K. & Deatline-Buchman, A. (2005). Effects of mathematical word problem-solving instruction on middle school students with learning problems. *The Journal of Special Education,* 39, 181–192. DOI: 10.1177/00224669050390030501

Appendix A

Participant Consent Form

Johns Hopkins University
Homewood Institutional Review Board (HIRB)
Key Informant Consent Form

Title: Examining the achievement gap between students without and with mathematical learning disabilities (MLD).

Principal Investigator: Dr. Christine Eith

Date: March 30, 2015

PURPOSE OF RESEARCH STUDY:
The purpose of the needs assessment study is to investigate the factors and underlying causes associated with the achievement gap between students without and with mathematical learning disabilities (MLD) from low socioeconomic households.

PROCEDURE:
There will be three components for this study:

1. The principal investigator will conduct an informal, semi-structured, and one-to-one interview with you – key informant.
2. Your responses to the interview questions will be captured via note taking and/or audio recording.
3. The principal investigator will analyze your responses.

Time Required for Interview: 30 minutes

RISKS/DISCOMFORTS:
There are no anticipated risks to key informants.

BENEFITS:
The potential benefits include:

1. Increased understanding of how teachers and parents can support students with MLD from low socioeconomic families.
2. Increased understanding of the factors associated with underachievement of students with MLD.
3. Academic achievement of students with MLD.

VOLUNTARY PARTICIPATION AND RIGHT TO WITHDRAW:

Your participation in this needs assessment study is entirely voluntary. If you want to withdraw or stop participating in the study, please contact Emmanuel Taiwo via phone or email: (240) 706-6042, etaiwo1@jhu.edu.

CONFIDENTIALITY:
Any study records that identify you will be kept confidential to the extent possible by law. Data may be reviewed by people responsible for making sure that research is done properly, including members of the Johns Hopkins University Homewood Institutional Review Board and officials from government agencies such as the Office for Human Research Protections. All of these people are required to keep your identity confidential.

COMPENSATION:
You will not receive any payment or other compensation for participating in the needs assessment study.

IF YOU HAVE QUESTIONS OR CONCERNS:
You can ask questions about this research study at any time during the study by contacting Emmanuel Taiwo via phone or email: (240) 706-6042, etaiwo1@jhu.edu. If you have questions about your rights as a research participant, please call the Homewood Institutional Review Board at Johns Hopkins University at (410) 516-6580.

CONSENT:
- I have read and understood the purpose of the research.
- I understand that my participation in this interview is voluntary.
- I have the right to not answer any question I don't like or to stop the interview and withdraw my answers, at any stage of the interview, without having to explain why.
- I understand that what I say will be kept confidential by the researchers and will only be used for research purposes. My name will not be used in any research reports and nothing will be published that might identify me.
- I understand that if I have any further questions I can contact one of the researchers listed on the information sheet
- I agree to the interview being audio recorded YES / NO
- I agree to some of my comments or statements being quoted in the report, provided that I cannot be identified YES / NO
- I would like to receive an edited copy of my interview transcript YES / NO
- I would like to receive a summary of the key findings from this study YES / NO

DECLARATION:
I, ________________________________ agree to be interview for this needs assessment study.

Signed/Date: ________________________ (Participant)

Signed/Date: ________________________ (Researcher)

Appendix B

Demographic Characteristics of Key Informants

Characteristics	Frequency	Percentage
Gender		
Female	7	87.5%
Male	1	12.5%
Age		
20-30	1	12.5%
30-40	1	12.5%
40-50	6	75.0%
Ethnicity		
White	7	87.5%
Black	1	12.5%

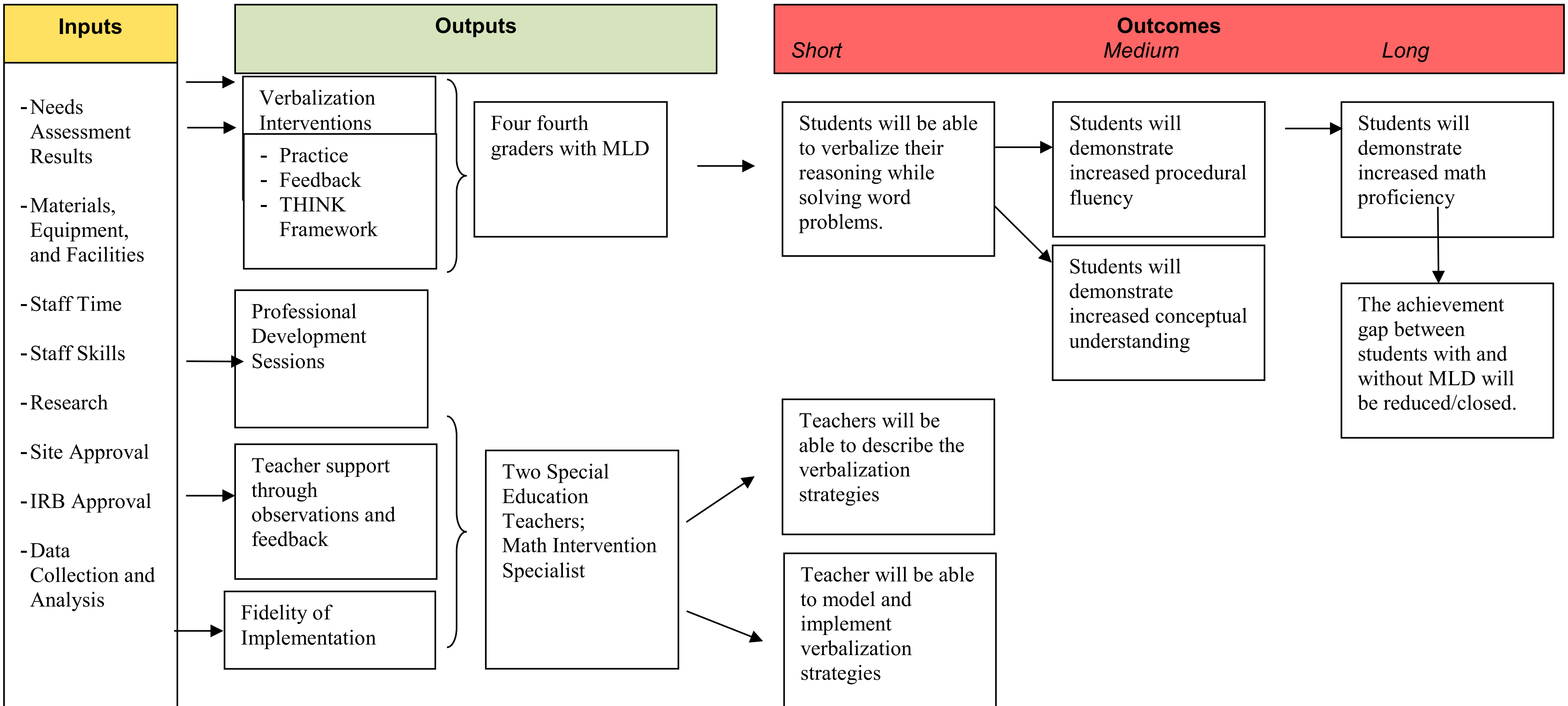
Appendix C

Codes and Categories

Category	Code
Cognition	▪ Memory ▪ Weak ▪ Processing ▪ Slow ▪ Reasoning ▪ Conceptual Understanding
Instructional Practices	▪ Training ▪ Multisensory ▪ Feedback ▪ Ineffective strategies
Socioeconomic	▪ Poverty ▪ Parental support ▪ Minimal parental involvement
School	▪ Common Core ▪ Low Expectations ▪ Unqualified Teachers ▪ Co-teaching models

Appendix D

Logic Model



Assumption
- Verbalization strategies explicitly modeled by teacher and subsequently used by students with MLD would improve math achievement.

External Factors
-Student absence or fatigue
- Parent disapproval
- Teacher withdrawal from the study
- Lack of fidelity of implementation

Rev. 7/09

Appendix E

Oral Consent Script

Application No.: HIRB00005246
Principal Investigator: Dr. Pare-Blagoev
Student Investigator: Emmanuel Taiwo
Date: 12/05/2016

ORAL ASSENT SCRIPT

I want to talk to you about a research study that I am doing. A research study is a way to learn information about something. I would like to find out more about an effective way of teaching mathematics to children who have a hard time when they learn and solve word problems. I am asking you to join the study to help us understand more about how to help children who sometimes struggle when they work on math.

If you agree to join this study, you will be asked to participate in several mathematics lessons. During these lessons, a teacher will be supporting you to do different math activities. You will be asked to talk out loud and say what you are thinking while solving math problems. Talking out loud like this while you work on math is called verbalizations.

This study will not cause you any harm or discomfort beyond what you might sometimes feel when you work on math problems. I do not know if you will be helped by being in this study but it is possible that the strategies you learn can be useful to you. I might learn something that will help other children with math difficulties someday.

You do not have to join this study. It is up to you. You can say okay now, and you can change your mind later. All you have to do is tell me. No one will be mad at you if you change your mind.

Before you say **yes** to joining this study, I will answer any questions you have. Before you answer, you can also talk with your parents.

Appendix F

IRB Approval

JOHNS HOPKINS
UNIVERSITY

Homewood Institutional Review Board
3400 N. Charles Street
Baltimore MD 21218-2685
410-516-6580
http://web.jhu.edu/Homewood-IRB/

Michael McCloskey, PhD
Chair

Date: March 7, 2017

PI Name: Elizabeth Pare-Blagoev
Study #: HIRB00005246
Study Name: Investigating the Effect of Student Verbalizations on the Mathematical Problem Solving of Fourth Grade Students with Mathematical Learning Disabilities (MLD)

Date of Review: 3/3/2017
Date of Approval: 3/3/2017
Expiration Date: 3/2/2018

The above referenced study has been *approved*.

Review Type:	Expedited
Funding Agency:	Not funded
Grant or Contract Number:	
International Sites:	No
Maximum number of participants:	Four students with MLD.
Vulnerable populations:	Children Homeless or Economically Disadvantaged
Consent process:	None
Assent Process:	Oral assent or Waiver of written documentation of assent Written parental permission

No changes may be made to the protocol or the consent form without the approval of the Board. Federal regulations require review of approved research not less than once a year, unless a shorter period is determined by the IRB. Therefore a Continuing Review progress report must be submitted no later than six weeks prior to the Study Expiration date of 3/2/2018 or within 30 days of study completion.

If continuing review approval is not granted before the expiration date of 3/2/2018 approval of this research expires on that date. Failure to submit a Continuing Review Progress Report prior to the approval lapse date will result in termination of the study, at which point new participants may not be enrolled and currently enrolled participants must discontinue participation in the study. All ongoing research activities must stop immediately, including data analysis.

Please keep in mind that it is your responsibility to inform the HIRB of any adverse consequences to participants that occur in the course of the study, as well as any complaints from participants regarding the research. In conducting this research, you are required to follow the requirements listed in the *HIRB Policies and Procedures Manual*.

Approved Documents:

Oral Assents:
Student Assent

Parental Permissions:
Parental Consent

Recruiting Materials:
OutreachRecruitmentPhoneGuide

Study Team Members:
Emmanuel Taiwo

APPROVAL IS GRANTED UNDER THE TERMS OF **FWA00005834** FEDERAL-WIDE ASSURANCE OF COMPLIANCE WITH DHHS REGULATIONS FOR PROTECTION OF HUMAN RESEARCH SUBJECTS

Appendix G

Recruitment Script

Recruitment Script:

Hello Mr./Ms.______

My name is Ms. _________, a 2nd-grade teacher at from Capital City Public Charter School. I am calling to inform you about a research study that one of our staff is doing. The research will explore ways to better teach children who struggle in math and we would like to know if you would like your child to participate. The research study includes several weeks of math intervention that will happen during Morning Math or Afterschool Intensives period, depending on your choice. During this time, a teacher will be assigned to support your student to learn new math strategies. If your student participates, he/she will be taught how to talk out loud about their mathematical thinking as they solve math problems.

Your child does not have to join this study. It is up to you. You can say yes/no now, and if you say yes, you can change your mind later. I would like to answer any questions you might have about the study to help you decide. Even after the study has begun, you can still decide that you do not what your student to continue to participate. Whether you decide to have your student participate or not is completely up to you and there is no penalty for your or your student if you decide not to participate.
Do you have any questions about the study?
At this point if there are questions they will be answered.

Are you interested in having your student participate and/or would you like more information?
If the parent responds no, at this point they will be thanked for their time and the call will end.

If the parent responds yes, additional information will be provided.

Thank you for your interest in having your child participate. The next step is for you to come in to school for a face to face meeting with Mr. Taiwo, the Director of Student Services. This study will be part of his dissertation. Mr. Taiwo will contact you to schedule a meeting at your convenience. At the meeting, you can ask more questions and Mr. Taiwo will be reviewing a consent form that describes more about the research. If you would like to reach out to Mr. Taiwo about the study, he can be reached at 202-808-9800 or etaiwo@ccpcs.org.

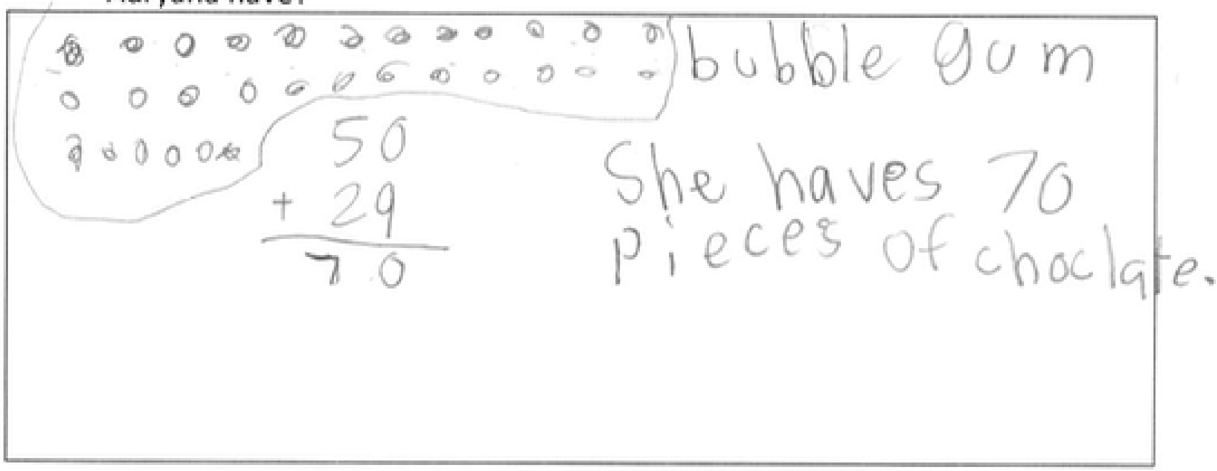
Do you have any other questions before I let Mr. Taiwo know that he should contact you about a meeting time?

If there are questions they will be answered. Once there are no other questions the parent/caregiver will be thanked for their time and interest and the call will end.

Appendix H

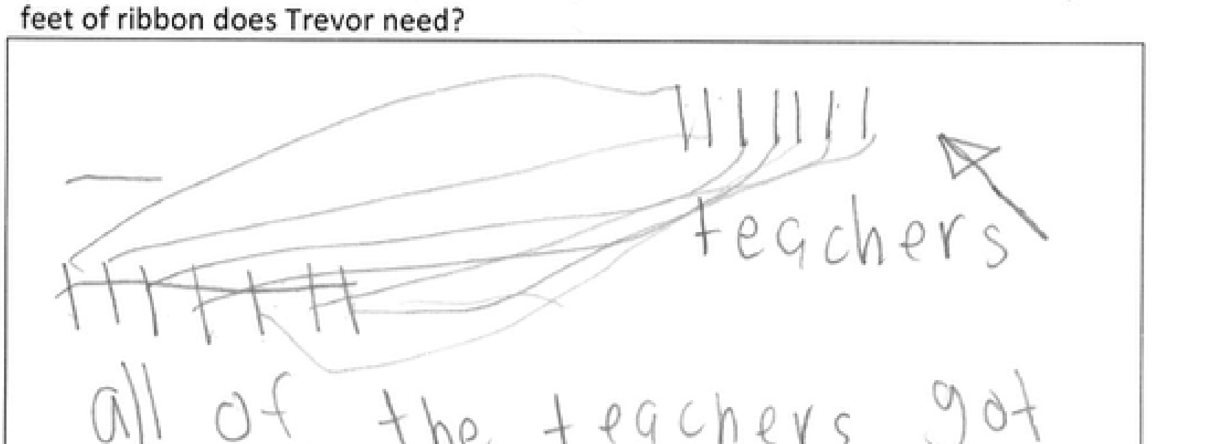
Selected Work Sample with Scores

Adryana has 50 pieces of candy. 29 of the pieces of candy are bubble gum and the rest are chocolate. How many pieces of chocolate candy does Adryana have?



Score: Understand __1__ Plan __1__ Solve __1__ Check __1__ = Total __1__

Trevor is helping his mom wrap gifts for his teachers. It is Trevor's job to cut the ribbon. It takes 3 feet of ribbon for each gift. Trevor has 7 teachers. How many feet of ribbon does Trevor need?



Score: Understand __1__ Plan __1__ Solve __1__ Check __1__ = Total __1__

Maria has 12 feet of ribbon and wants to wrap some gifts that need 3 feet of ribbon each. How many gifts can she wrap using the ribbon?

TALK about the problem. • What do you know about the problem? • Explain what the problem is asking you to do.	12 ft. 卌| = 3ft
HOW can the problem be solved? • What are the strategies you can use to solve the problem? • Talk about how to use your strategy.	division
IDENTIFY and USE a strategy for solving the problem. • Use the strategy.	12÷3 = 3 12÷3=4
NOTICE how your strategy helped you solve the problem. • Is your strategy working, or do you need to choose another one to solve the problem? • Why did you solve tit this way?	I youced Division
KEEP thinking about the problem. • Check your answer. • Does your answer make sense? • Why do you think your solution is correct and make sense?	4 she can wrap 4 gifts it make sense.

Score: Understand __3__ Plan __2__ Solve __2__ Check __2__ = Total __2.25__

Kaleb and Ethan are building towers with blocks. Kaleb's tower has 23 blocks. Ethan's tower has 45 blocks. How many blocks will there be if they put both towers together to make one big tower?

TALK about the problem. • What do you know about the problem? • Explain what the problem is asking you to do.	Kaleb: 23 blocks Ethan = 45 blocks one big tower
HOW can the problem be solved? • What are the strategies you can use to solve the problem? • Talk about how to use your strategy.	23 + 45 =
IDENTIFY and USE a strategy for solving the problem. • Use the strategy.	 68 blocks
NOTICE how your strategy helped you solve the problem. • Is your strategy working, or do you need to choose another one to solve the problem? • Why did you solve tit this way?	I ues unifix cubes and I put they to ther. to make 68 Blocks.
KEEP thinking about the problem. • Check your answer. • Does your answer make sense? • Why do you think your solution is correct and make sense?	also I will have my grat like 68.

Score: Understand __3__ Plan __3__ Solve __3__ Check __3__ = Total __3__

Deryn's bookshelf has 3 shelves with 7 books on each shelf. How many books are on the bookshelf?

TALK about the problem. • What do you know about the problem? • Explain what the problem is asking you to do.	3 Shelves 7 Books on each Shlf
HOW can the problem be solved? • What are the strategies you can use to solve the problem? • Talk about how to use your strategy.	Addition or multiplication 7+7+7=
IDENTIFY and USE a strategy for solving the problem. • Use the strategy.	21 Book on the Book Shelf
NOTICE how your strategy helped you solve the problem. • Is your strategy working, or do you need to choose another one to solve the problem? • Why did you solve tit this way?	Yes it help and I Will ves it a agned
KEEP thinking about the problem. • Check your answer. • Does your answer make sense? • Why do you think your solution is correct and make sense?	BecauseI understad my Work and it make Sense

Score: Understand __3__ Plan __3__ Solve __3__ Check __3__ = Total __3__

A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times longer is the rubber band than it was at first?

TALK about the problem. • What do you know about the problem? • Explain what the problem is asking you to do.	6 cm 18 cm
HOW can the problem be solved? • What are the strategies you can use to solve the problem? • Talk about how to use your strategy.	18 ÷ 6 =
IDENTIFY and USE a strategy for solving the problem. • Use the strategy.	3 times longer
NOTICE how your strategy helped you solve the problem. • Is your strategy working, or do you need to choose another one to solve the problem? • Why did you solve tit this way?	I ues Unifix cuber I Put tham in 3 and They have 6.
KEEP thinking about the problem. • Check your answer. • Does your answer make sense? • Why do you think your solution is correct and make sense?	I had 18 and then I have one and Then tow and Theer

Score: Understand __3__ Plan __3__ Solve __3__ Check __3__ = Total __3_

Appendix I

Types of Problems Used in Each Session

Session Item Introduced: A1/B1 Initial Intervention	Session Item Introduced: A2/B2 Delayed Intervention	Item	Common Core State Standard	Grade Level
		Baseline Items		
S1	S1	Colby has 73 baseball cards, and Jack has 59 baseball cards. How many fewer baseball cards does Jack have than Colby?	2.OA.A.1	2
S2	S2	John read 28 more pages than Gary. Gary read 95 pages. How many pages did John read?	2.OA.A.1	2
S3	S3	Emarion arranged his toy soldiers in straight rows. He made 6 rows with five soldiers in each row. How many toy soldiers did Emarion have?	3.OA.A.3	3
S4	S4	Chris had 48 candy bars to sell. He sold some of the candy bars and now has 24 candy bars. How many candy bars did Chris sell?	2.OA.A.1	2
S5	S5	Trevor is helping his mom wrap gifts for his teachers. It is Trevor's job to cut the ribbon. It takes 3 feet of ribbon for each gift. Trevor has 7 teachers. How many feet of ribbon does Trevor need?	3.OA.A.3	3
S6	S6	Sydney's mom baked 12 cookies. Sydney's sister then baked some more cookies. Now there are 30 cookies. How many cookies did Sydney's sister bake?	2.OA.A.1	2
N/A	S7	A pencil costs 59 cents, and a sticker costs 23 cents less. How much do a pencil and a sticker cost together?	2.OA.A.1	2
N/A	S8	A carnival is in town for 21 days. How many weeks is the carnival in town? There are 7 days in 1 week.	3.OA.A.3	3
N/A	S9	Alexa is practicing for a race. She ran for 35 minutes on Friday and 47 minutes on Saturday. How much longer did Alexa run on Saturday?	2.OA.A.1	2
N/A	S10	Aunt Korina and her 3 friends decide to share a cab to go to the mall. If they	3.OA.A.3	3

		each spent $6, how much did the cab ride cost altogether?		
		Intervention Items		
S7	S11	Melanie had some nickels in her bank. After saving for a long time, Melanie added 27 more nickels to her bank. Her bank now has 40 nickels in it. How many nickels did Melanie begin with?	2.OA.A.1	2
S8	S12	A chef is cooking chicken in a restaurant. The recipe says you need 5 minutes for every pound. How many minutes will it take to cook 12 pounds of chicken?	3.OA.A.3	3
S9	S13	Kim has 75 dollars in the bank. She spent 38 dollars. How many dollars does Kim have in the bank now?	2.OA.A.1	2
S10	S14	Nina can practice a song 6 times in an hour. If she wants to practice the song 30 times before the recital, how many hours does she need to practice?	3.OA.A.3	3
S11	S15	Molly baked some cupcakes for a bake sale. She sold 35 and had 37 left over. How many cupcakes did Molly bake?	2.OA.A.1	2
S12	S16	Maria has 12 feet of ribbon and wants to wrap some gifts that need 3 feet of ribbon each. How many gifts can she wrap using the ribbon?	3.OA.A.3	3
S13	S17	Each basketball team has 5 players on it. There are 30 players in a league. How many teams are playing in the league?	3.OA.A.3	2
S14	S18	Anna had 72 tickets for the carnival rides. She used 10 tickets for the roller coaster and 12 tickets for the rocket ride. How many tickets does Anna have now?	2.OA.A.1	3
S15	S19	Aubrey had a box of crayons. She found 14 more crayons when she cleaned out her desk and put them in the box. Now there are 47 crayons in the box. How many were in the box to begin with?	2.OA.A.1	2
S16	S20	Deryn's bookshelf has 3 shelves with 7 books on each shelf. How many books are on the bookshelf?	3.OA.A.3	3
S17	S21	A rubber band was 6 cm long at first.	3.OA.A.3	3

		Now it is stretched to be 18 cm long. How many times longer is the rubber band than it was at first?		
S18	S22	Kaleb and Ethan are building towers with blocks. Kaleb's tower has 23 blocks. Ethan's tower has 45 blocks. How many blocks will there be if they put both towers together to make one big tower?	2.OA.A.1	2
N/A	S23	Cole and Maggie worked together to make a paper chain with 85 links. At the end of the day, they each wanted to take home part of the chain. The part Maggie took had 43 links. How many links were in Cole's part?	2.OA.A.1	2
N/A	S24	There were 24 people in a marching band. They lined up in equal rows. Show how they could have lined up. Write an equation that represents your drawing.	3.OA.A.3	3
N/A	S25*	Graham had 45 baseball cards in his collection. He bought 23 more at his neighbor's yard sale. How many baseball cards does Graham have now?	2.OA.A.1	2
N/A	S26*	Penny has 63 party favors to give to 7 friends. She wants to give each friend the same amount. How many party favors should she give to each friend?	3.OA.A.3	3

Note. Appendix I shows the word problem students completed independently at the end of each session and analyzed using Thomas's (2006) problem solving scoring rubric. 2.OA.A.1 required students to use addition and subtraction within 100 to solve one-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. 3.OA.A.3 required students to use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.

*Student B2 did not participate in sessions 25 and 26 due to illness during the last week of school.

Curriculum Vitae

Emmanuel Taiwo
6656 Waning Moon Way, Columbia, MD 21045
etaiwo1@jhu.edu, (240) 706-6042

EDUCATION

Johns Hopkins University, Baltimore, MD Expected Dec. 2017
Doctor of Education (Ed.D.) in Special Education

Lesley University, Cambridge, MA May 2013
Master of Education (M.Ed.) in Special Education

Lesley University, Cambridge, MA May 2013
Master of Education (M.Ed.) in Elementary Education

Obafemi Awolowo University, Nigeria December 2007
Bachelor of Science (B.S.) in Physics Education

Research Interest

- Interventions to improve the mathematical performance of students with disabilities.
- Special education teacher training and professional development.

Research Experience

Doctoral Research 2013-2017
Department of Special Education, Johns Hopkins University

- Conducted a needs assessment study to investigate the achievement gap of children with mathematical learning disabilities enrolled at a public charter school in an east coast metropolitan area.
- Conducted a dissertation study to determine the effectiveness of verbalizations on the mathematical problem solving of four fourth grade students with MLD.

TEACHING AND MENTORING EXPERIENCE

Johns Hopkins University School of Education/Urban Teachers, DC **August 2017 – Present**

Adjunct Instructor

- Plan and facilitate class and online instruction for graduate students.
- Ensure that the syllabus meets department standards.
- Grade assigned papers, quizzes and exams.
- Assess grades for students based on participation, performance in class, assignments and examinations.
- Collaborate with colleagues on course curriculum.

Capital City Public Charter School – Washington, DC **July 2014 – Present**
Director of Student Services

- Coordinate special education program (pre-referral interventions, evaluation/eligibility process, service delivery, program evaluation) for children with disabilities;
- Review, analyze, and report performance measures to inform programmatic changes.
- Responsible for special education compliance, state reporting, data collection, review, and day-to-day challenges that arise. Prepare and maintain proper records and reports using systems such as SEDS (Easy-IEP), SLED, DC CATS, etc.
- Maintain and improve systems and procedures related to special education, English language learner (ELL) program, behavior support, and RtI.
- Design and lead professional development sessions for teachers and on evidence-based interventions and best practices for educating students with disabilities.
- Coordinate the English learner program (identification, assessment, placement, and services).
- Coordinate and manage the administration of ACCESS for ELLs 2.0.
- Establish partnerships with external stakeholders to enhance the inclusion program.

Capital City Public Charter School – Washington, DC **August 2012 – July 2014**
Special Education Teacher

- Implemented evidence-based strategies/interventions for elementary children with disabilities.
- Collaborated with general education teachers to plan and deliver instruction using different co-teaching models.
- Provided specialized instruction and implemented research-based intervention programs such as Wilson Reading System (WRS), Fountas and Pinnell Leveled Literacy Intervention (LLI), On Cloud Nine® Math Program and others.
- Assumed responsibility for the timely and legal management of caseload student IEPs.
- Collaborated with the school psychologist to conducted Functional Behavior Assessment (FBA) and developed/implemented Behavior Intervention Plan (BIP).
- Collected and analyzed student data to improve instruction for children with disabilities.

Septima Clark Public Charter School – Washington, DC **August 2011 – July 2012**
3rd Grade Teacher

- Differentiated mathematics and science instruction to meet the needs of all learners.
- Provided student-centered, interactive, hands-on learning experiences that foster discovery, creativity, problem solving, collaboration, and student choice.
- Maintained data systems to monitor students' learning.

Christ Academy International School – Abuja, Nigeria **August 2008 – July 2011**
Mathematics and Science Teacher

- Created and implemented lesson plans.
- Assess students' performance and monitor their progress.
- Communicated with parents about their child's progress.
- Worked with students individually to help them overcome specific learning challenges.

CONFERENCE PRESENTATIONS

POSTER PRESENTATIONS Emmanuel Taiwo. (2017, August). The effect of student verbalizations on the mathematical problem solving of fourth grade students with mathematical learning disabilities (MLD).

CERTIFICATION AND PROFESSIONAL DEVELOPMENT

- DC Regular II license - Special Education Specialization (Non-Categorical), Grades K-12
- DC Regular II license - Elementary, Grades 1-6
- World-class Instructional Design and Assessment (WIDA)
- Wilson Reading System® (WRS) - Level 1 Certification
- On Cloud Nine® Math Program
- Fountas and Pinnell Leveled Literacy Intervention (LLI)
- Positive Behavior Interventions and Supports (PBIS) and Responsive Classroom Approach
- Center for Transformative Teacher Training, No-Nonsense Nurturer Workshop
- Expeditionary Learning Education National Conventions and Site Seminars
- Dynamic Indicators of Basic Early Literacy Skills (DIBELS)
- Center for Applied Linguistics (CAL)
- Council for Exceptional Children (CEC) Convention

LANGUAGES

- English: Proficient
- Yoruba: Proficient

REFERENCES

E. Juliana Paré-Blagoev, Assistant Professor
Johns Hopkins University
School of Education
juliana.pareblagoev@jhu.edu

Karen Karp, Visiting Professor
Johns Hopkins University
School of Education
kkarp1@jhu.edu

Aaron Parsons, Vice President
K-12 School Programs at Kennedy Krieger Institute
The Johns Hopkins University
parsons@kennedykrieger.org